Home Exercises 2

Your Name

2.10.2023

Write your name at the beginning of the file as “author:”.

1. Return to Moodle by **9.00am, Mon 2.10.** (to section “BEFORE”).
2. Watch the exercise session video available in Moodle by **10.00am, Mon 2.10.**
3. If you observe during the exercise session that your answers need some correction, return a corrected version to Moodle (to section “AFTER”) by **9.00 am, Mon 9.10.**

Problem 1. Suppose that you treat $n = 97$ patients with a new treatment and 70 patients benefit. You know that the old treatment helped about 70% of the patients. Your null hypothesis is that also the new treatment has success probability of 70%.

(a) Plot the null distribution Bin(97, 0.7) using continuous line connecting the probability values for range 0 to 97 successes. Mark the observed value of $x = 70$ by a red line. (See Example 2.3. from lecture notes.) What do you conclude about how consistent the observation is with the null hypothesis by simply looking at the plot? Would you expect to have a small P-value in these data?

(b) Compute a right-hand tail probability of observing at least 70 successes under the null hypothesis. How could you use this value to approximate the **2-sided** P-value? (See Example 2.3 from lecture notes.)

(c) Compute the exact two-sided P-value under the null hypothesis for your observation using `binom.test()`.

(d) Do you think that the new treatment may be more efficient than the old one?

Problem 2. Let’s move back in time to the beginning of COVID-pandemic. In January 2020, Lancet published this article:

Huang et al. (2020) Clinical features of patients infected with 2019 novel coronavirus in Wuhan, China. Lancet 395: 497-506. https://www.thelancet.com/journals/lancet/article/PIIS0140-6736(20)30183-5/fulltext

It described 41 confirmed COVID-cases from Wuhan, China. This small study gives good examples to study Fisher’s exact test since these small counts are too small for many other tests. In the following questions, we compare properties of patients taken to intensive care unit (ICU) and those outside ICU.

(a) Among ICU patients, 11 were men and 2 women, while among non-ICU patients, 19 were men and 9 were women. Use Fisher’s test to evaluate whether there is a statistical association between ICU and sex. What is the P-value?

(b) Among 13 patients in ICU, 11 had acute respiratory distress syndrome while among 28 patients in non-ICU 2 had it. Is there an association between ICU and respiratory syndrome at significance level 0.001? (NOTE: Be careful to put the correct numbers into data matrix. They are not all directly given above.)

(c) Among 13 patients in ICU, 5 died while among 28 patients in non-ICU 1 died. Is there an association between ICU and death at significance level 0.01?

Problem 3. In Finland, on average, 8.1% of babies are born in April. Suppose that we have checked the month of birth for $N = 5000$ MS-disease patients and observed that $A = 485$ of them were born in April. Our interest is whether being born in April is a risk factor for MS-disease.

(a) Is the proportion of MS-patients who were born in April larger or smaller than the population frequency of being born in April?

(b) Suppose as a null hypothesis that MS-disease patients were just as probable to be born in April as general population. Then the value A that we have observed would come from the distribution Bin(5000, 0.081) (5000 trials and success probability is 0.081 for each trial). Generate 10000 experiments where each takes a sample of `size=5000` individuals from binomial distribution with `prob=0.081`. Generate histogram of the results. (See use of `rbinom()` from Lecture 1.)

(c) Look visually where the observed value $A = 485$ would lie in the histogram of part (b). Does it seem a plausible value according to Bin(5000, 0.081) distribution? Or does it seem to be smaller or larger than one might expect if Bin(5000, 0.081) held true for MS-patients?

(d) Compute probability that a value from Bin(5000, 0.081) is at least 485 using `pbinom()`?

(e) Do a binomial test to compute a (two-sided) P-value for observation 485 successes out of 5000 trials with success probability of 0.081.

Problem 4. Let's try to find an intuitive way to quantify how "surprising" any given P-value is by utilizing coin flipping experiments.

Let's think about experiment of flipping n coins. The null hypothesis is that the coins are fair (success probability of heads is 50%). Let's think that we observed all n coins landing heads up. If there is only one coin, then observing heads is not surprising at all (probability $1/2 = 50\%$). If there are two coins and both land heads up, that's not very surprising either (probability $1/4 = 0.25$). But if ten coins land heads up, most of us would start doubt the null hypothesis of fair coins since probability of this event is only 1/1024 or 0.00098. On the other hand, we can now interpret that observing, for example, P-values of 0.5, 0.25 or 0.00098 correspond to a similar amount of surprise under the null hypothesis, than observing 1,2 or 10 coins landing heads up when we assumed that they were all fair coins.

Probability of observing only heads from n coins is $1/2^n$. If we want to match our observed P-value P to the number of coins n that would correspond to the same amount of surprise when they all have landed heads up, we have $P = 1/2^n$ or equivalently $n = -\log_2(P)$. Thus, for any P-value `P`, the corresponding number of coins landing heads up can be computed in R by command `-log2(P)`. This quantity is also called S-value or surprise value.

(a) The most common significance threshold used in medical literature is 0.05. What is the S-value corresponding to P-value 0.05? Do you feel that P-value of 0.05 is very surprising under the null hypothesis based on the S-value?

(b) In genome-wide scans of risk variants, a significance threshold of 5e-8 is used. What is the corresponding S-value and how surprising such an observation seems to you?

(c) In Problem 2, we encountered a P-value of 1.694e-06. What is the corresponding S-value and how surprising this P-value feels to you?