COSMOLOGY Homework 9 Fall 2018

Due on December 1 by 14.00.

1. **Free massless scalar field.** Consider a spatially flat universe dominated by a scalar field with $V = 0$. Find the scale factor and the scalar field as a function of time.

2. **Slow-roll parameters.**

   a) Demonstrate that $\varepsilon(\varphi) \ll 1$ and $|\eta(\varphi)| \ll 1$ are necessary conditions for the slow-roll approximation to be valid.

   b) Explain why these conditions are not sufficient.

3. **E-folds of inflation.**

   a) Derive eq. (8.48) of the lecture notes.

   b) Consider the inflation model with with the potential $V(\varphi) = \frac{1}{2}m^2\varphi^2$. Assume that $m = 2 \times 10^{13}$ GeV. Take inflation to end when $\varepsilon = \eta = 1$ and find the value of φ at the end of inflation. Then find the value of ε and η when $N = 50$. What scale k (in units of H_0^{-1}) does this value of N correspond to, assuming instant reheating?

4. **Bonus problem: spatial curvature.** This problem is worth six extra points (not counted against the maximum points from homework). Assume that at the beginning of inflation we have $|\Omega_K| = 0.1$.

   a) Calculate, as a function of the reheating temperature T_{reh}, how many e-folds of inflation are required to reduce present-day spatial curvature to $|\Omega_{K0}| < 10^{-2}$. (Assume $h = 0.7$ and that neutrinos are massless.) Approximate that the expansion rate at the beginning of inflation is completely dominated by the inflaton, that the inflaton field value does not change during inflation and that reheating happens instantaneously.

   b) In which directions do the above approximations change the result?

   c) What is the number of e-folds for $T_{\text{reh}} = 10^7$ GeV?