Instructions. Each question is worth 6 points. Answer in English, Finnish, or Swedish. In *questions 1–2*, it is sufficient that you give an *informal description of the algorithm*—you do not need to use the precise state-machine formalism, and you do not need to prove that your algorithm is correct.

Definitions. An *independent set* of a graph $G = (V, E)$ is a subset of nodes $I \subseteq V$ such that there is no edge $\{u, v\} \in E$ with both $u \in I$ and $v \in I$.

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Question 1: Deterministic algorithms. Design a deterministic distributed algorithm that solves the following problem in time $O(\log n)$ in the LOCAL model:

- Graph family: cycle graphs.
- Local inputs: unique identifiers.
- Local outputs: a *maximal* independent set.

You can assume that the unique identifiers are integers between 1 and n. You can also assume that all nodes know n.

Question 2: Randomised algorithms. Design a randomised distributed algorithm that solves the following problem in the PN model:

- Graph family: cycle graphs.
- Local inputs: nothing.
- Local outputs: a *maximal* independent set.

Question 3: Covering maps. Prove that the following problem cannot be solved at all with deterministic distributed algorithms in the PN model:

- Graph family: cycle graphs.
- Local inputs: nothing.
- Local outputs: a *maximal* independent set.

You can use the following textbook result (without proving it): covering maps preserve local outputs.

Question 4: Local neighbourhoods. Prove that the following problem cannot be solved in time $o(n)$ with deterministic distributed algorithms in the LOCAL model:

- Graph family: cycle graphs.
- Local inputs: unique identifiers.
- Local outputs: a *maximum* independent set.

You can use the following textbook result (without proving it): isomorphic radius-T neighbourhoods imply identical local outputs for time-T deterministic algorithms.