Instructions. Each question is worth 6 points. You can answer in English, Finnish, or Swedish. In *question 1*, you are expected to give a normal *mathematical proof* (similar to what you would do if this was a math course). In *questions 2–4*, it is sufficient that you give an *informal description of the algorithm*—you do not need to use the precise state-machine formalism, and you do not need to prove that your algorithm is correct. However, please give enough details so that I can understand e.g. what messages your algorithm sends in each possible situation. Also please make sure that your answer is entirely self-contained; for example, if you want to use some algorithms from the course material as subroutines, you will also have to give the details of those algorithms.

Definitions. Recall that a *weak k-colouring* of a graph $G = (V, E)$ is a labelling $f : V \to \{1, 2, \dots, k\}$ of the nodes such that each non-isolated node u has a neighbour v with $f(u) \neq f(v)$. As usual, n denotes the number of nodes in the network.

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Question 1: Graph-theoretic foundations. Prove: For any graph $G = (V, E)$, there exists a weak 2-colouring of G.

Question 2: PN model. Give a deterministic distributed algorithm that solves the following problem in time $O(n)$ in the PN model:

- Graph family: path graphs with at least 3 nodes.
- Local inputs: nothing.
- Local outputs: a weak 2-colouring.

Question 3: LOCAL model. Give a deterministic distributed algorithm that solves the following problem in time $O(1)$ in the LOCAL model:

- Graph family: cycle graphs.
- Local inputs: nothing (except the unique identifiers).
- Local outputs: a weak k-colouring for some $k = O(\log n)$.

Question 4: CONGEST model. Give a deterministic distributed algorithm that solves the following problem in time $O(n)$ in the CONGEST model:

- Graph family: cycle graphs.
- Local inputs: nothing (except the unique identifiers).
- Local outputs: each node outputs the set of all unique identifiers.

For example, if there are 4 nodes, and they are labelled with identifiers 3, 7, 10, and 12, then all nodes have to output the set $\{3, 7, 10, 12\}$.