Logical Characterizations in Distributed Computing

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Recently in Paris...

I was one of the examiners in Fabian Reiter's PhD defense at Paris Diderot

Fabian's talk started with ***Fagin's theorem*** and then proceeded to introduce the ***"Helsinki–Tampere theorem"***

What is this about?

Back to February 2010

I gave a talk in the **Finite Model Theory Seminar** on an unusual topic: **models of distributed computing**

Led to a collaboration that initiated the study of ***distributed graph algorithms*** from the perspective of ***descriptive complexity***

Helsinki–Tampere team

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What is distributed computing?

Centralized vs. distributed

- Theory of ***centralized*** computing:
  - “what can be computed efficiently with my laptop?”
  - input & output: encoded as a **string**
  - model of computing: **Turing machines**
- Theory of ***distributed*** computing:
  - ???

Distributed computing

- Can mean ***lots of different things***
  - causes lots of confusion
- I'll explain ***two commonly used interpretations***
  - these are just **"two extremes"**
  - there is a whole spectrum of variants between them

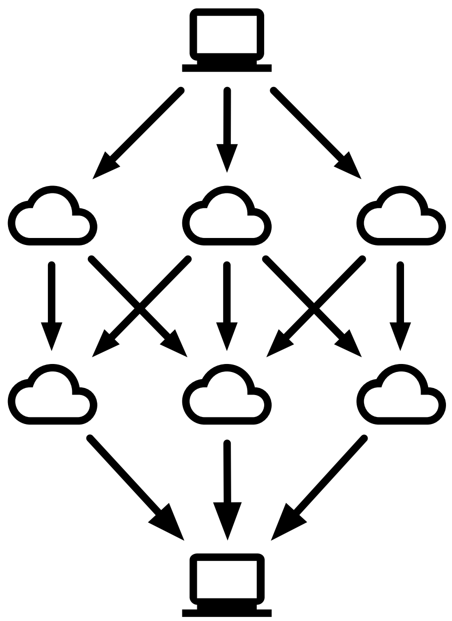
Big data perspective

"*Too large* for my laptop to solve, I'll have to resort to Amazon cloud"

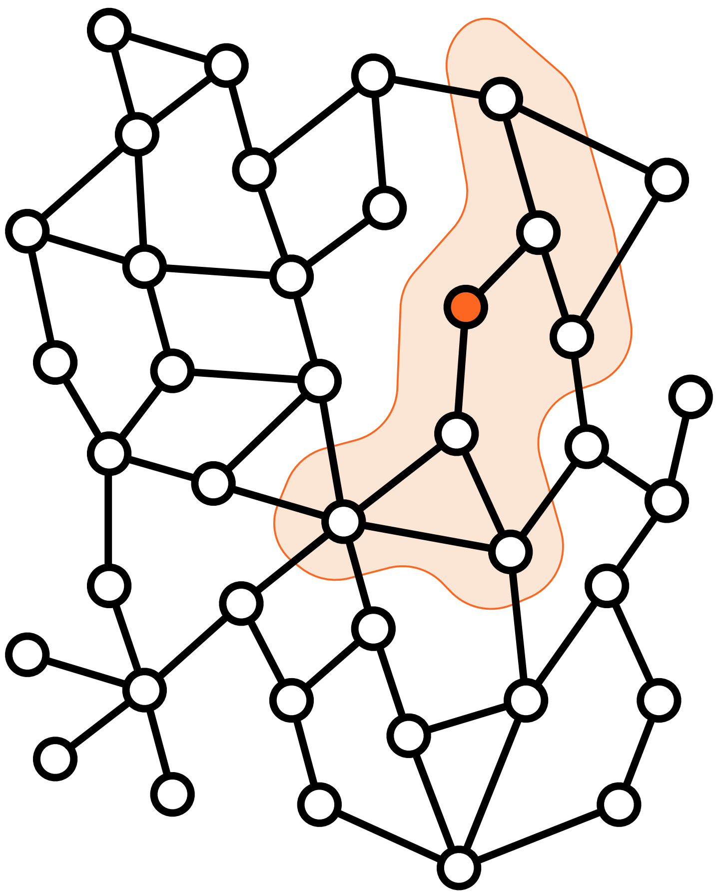
Network algorithms

"How to coordinate data transmissions in a large network ***without centralized control***?"

Big data perspective



Network algorithms



Big data perspective

- Focus: ***computation***
- Distributed perspective **helps** us

Network algorithms

- Focus: ***communication***
- Distributed perspective additional **challenge**

Big data perspective

- Fully centralized control
- *Global* perspective
- **Input & output in one place**

Network algorithms

- No centralized control
- *Local* perspective
- **Input & output distributed**

Big data perspective

- I know ***everything*** about input
- I need to know ***everything*** about solution

Network algorithms

- Each node knows its ***own part*** of input
  - e.g. local constraints
- Each node needs its ***own part*** of solution
  - e.g. when to switch on?

Big data perspective

- Explicit input
  - encoded as a string, stored on my laptop
- Well-known network structure
  - tightly connected cluster computer

Network algorithms

- Implicit input
  - ***input graph = network structure***
- Unknown network structure
  - e.g. entire global Internet right know

Big data perspective

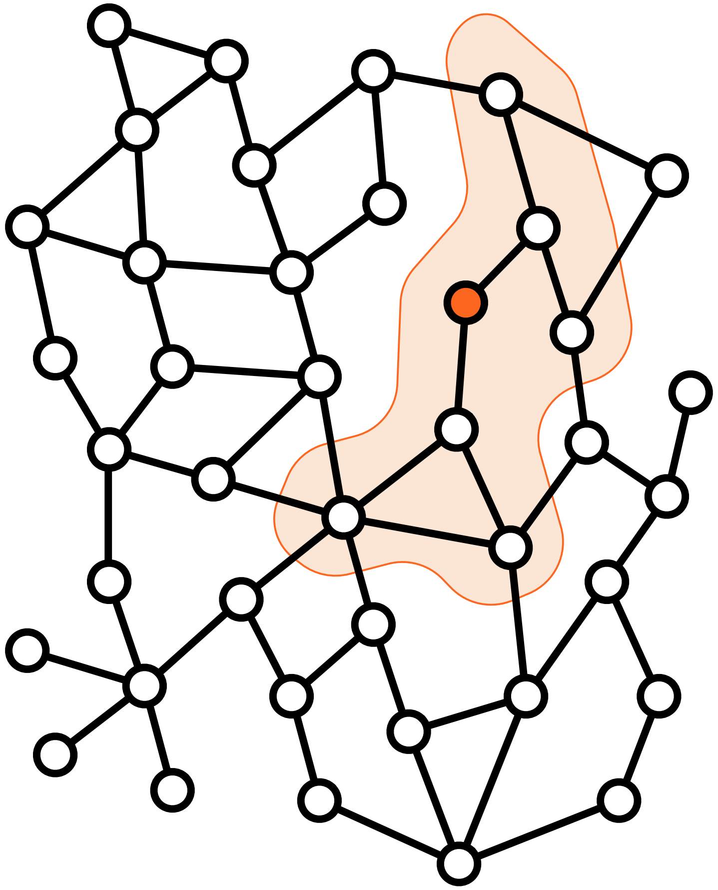
Can we divide problem in **small independent tasks** that can be solved **in parallel**?

Network algorithms

If each node is only aware of its **local neighborhood**, can we nevertheless find a **globally consistent solution**?

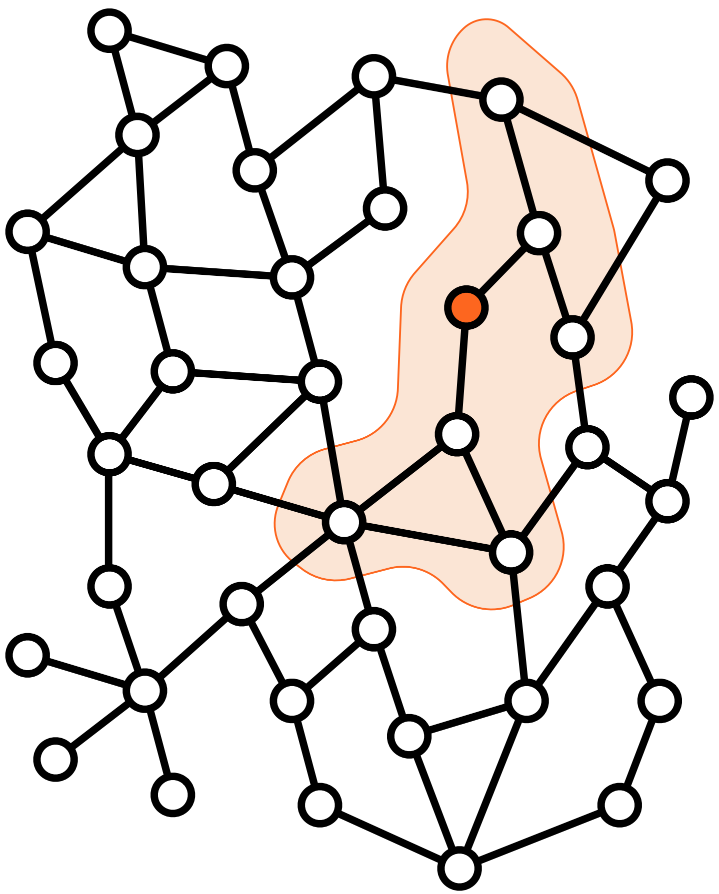
Big data perspective

Network algorithms



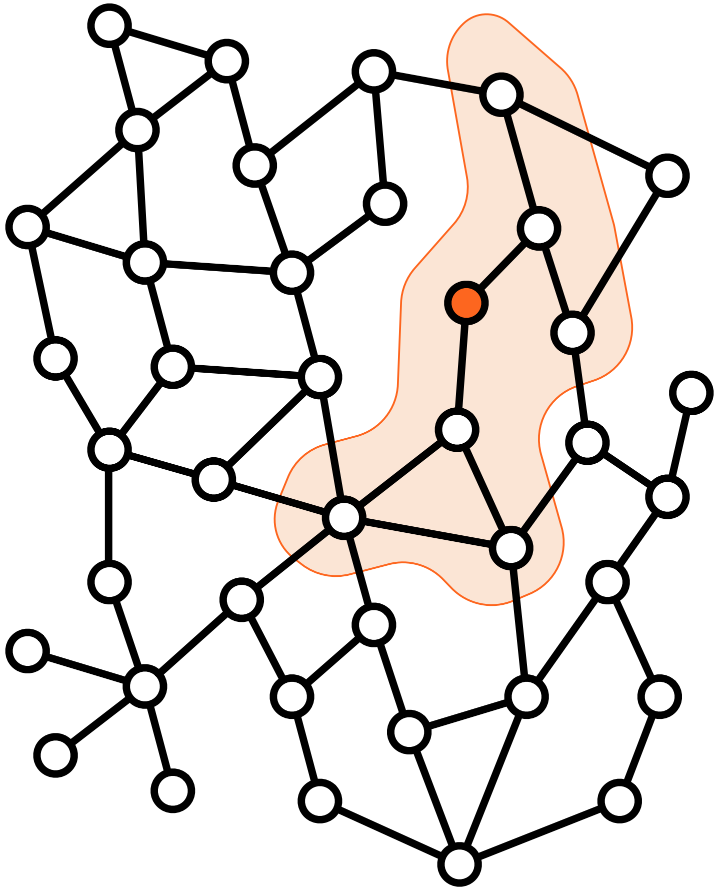
LOCAL model

- Initial knowledge:
  - local input, number of neighbors
- Communication round:
  - **send** message to each neighbor
  - **receive** message from each neighbor
  - **update** state
  - possibly: ***announce local output and stop***



LOCAL model

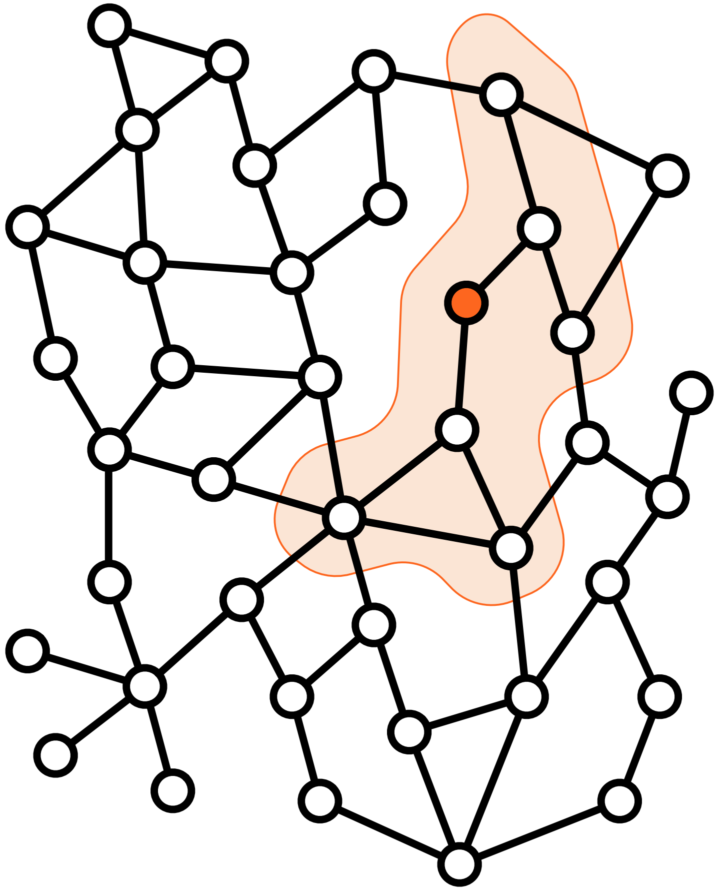
- Each node labeled with a **"unique identifier"**
  - constant k such that if we have a graph with n nodes, unique identifiers are distinct values from $\{ 1, 2, \ldots, n^k \}$



LOCAL model

Equivalent:

- "*running time*"
- number of synchronous ***communication rounds***
- ***how far*** do we need to look in the graph



Fast algorithm ↔ highly "localized" solution

LOCAL model

- The usual computer science perspective:
  - what is the ***worst-case running time***?
  - ***asymptotically***, as a function of n
- Two-player game:
  - **player A** chooses the **algorithm**
  - **player B** then chooses the **graph**, **local inputs**, **unique identifiers**

LOCAL model

- ***Everything is computable in $O(n)$ rounds!***
  - assuming a **connected graph**
  - gather everything, solve locally by brute force
  - exploits: large messages, unlimited local computation
- Interesting question: what can be done in ***$o(n)$ rounds***?

LOCAL model: examples

- Example: ***graph coloring*** with k colors
  - **local input:** nothing
  - **local output:** what is my own color
  - **constraint:** adjacent nodes have different colors

LOCAL model: examples

- Example: ***graph coloring*** with k colors
- Graph family: ***path*** with n nodes
  - $k = 2$: $\Theta(n)$ rounds
  - $k = 3$: $\Theta(\log^* n)$ rounds
  - $k = 100$: $\Theta(\log^* n)$ rounds

LOCAL model: examples

- Example: ***graph coloring*** with k colors
- Graph family: 2D ***grid*** with $n \times n$ nodes
  - $k = 2$: $\Theta(n)$ rounds
  - $k = 3$: $\Theta(n)$ rounds
  - $k = 4$: $\Theta(\log^* n)$ rounds
  - $k = 100$: $\Theta(\log^* n)$ rounds

LOCAL model: examples

- Example: ***weak 2-coloring***
  - label nodes with {0, 1}
  - each node has **a neighbor with a different label**
- Graph family: ***regular graphs***
  - 4-regular graphs: $\Theta(\log^* n)$ rounds
  - 5-regular graphs: $\Theta(1)$ rounds

LOCAL model

- Why do we keep seeing “$\Theta(\log^* n)$”?
- All of these are algorithms that exploit ***numerical values of unique identifiers***
  - more precisely, it is $\Theta(\log^* s)$, where s **= size of the identifier space**
  - we just assumed that $s = \text{poly}(n)$

LOCAL model

- What if we don’t have unique identifiers?

Weak models of distributed computing

“Weak models”

- Initial knowledge:
  - local input, number of neighbors
- Communication round:
  - **send** message to each neighbor
  - **receive** message from each neighbor
  - **update** state
  - possibly: ***announce local output and stop***

All of this identical to the LOCAL model!

“Weak models”

- Key difference: ***nodes are identical***
  - no unique identifiers
  - “anonymous networks”

“Weak models”

- How to refer to your neighbors?
- ***Port-numbering model:***
  - node of degree *d* can refer to its neighbors with numbers **1, 2, …, *d***
  - “this is the message that I got from neighbor *x*”
  - “I want to send this message to neighbor *x*”

“Weak models”

- How to refer to your neighbors?
- ***Set–broadcast model:***
  - no way to refer to specific neighbors
  - “this is the **set of messages** that I got from my neighbors in this round”
  - “I want to **broadcast** this message to all neighbors”

Weak models: computability

- Many problems cannot be solved at all
- Key challenges:
  - breaking symmetry
  - detecting cycles

Breaking symmetry

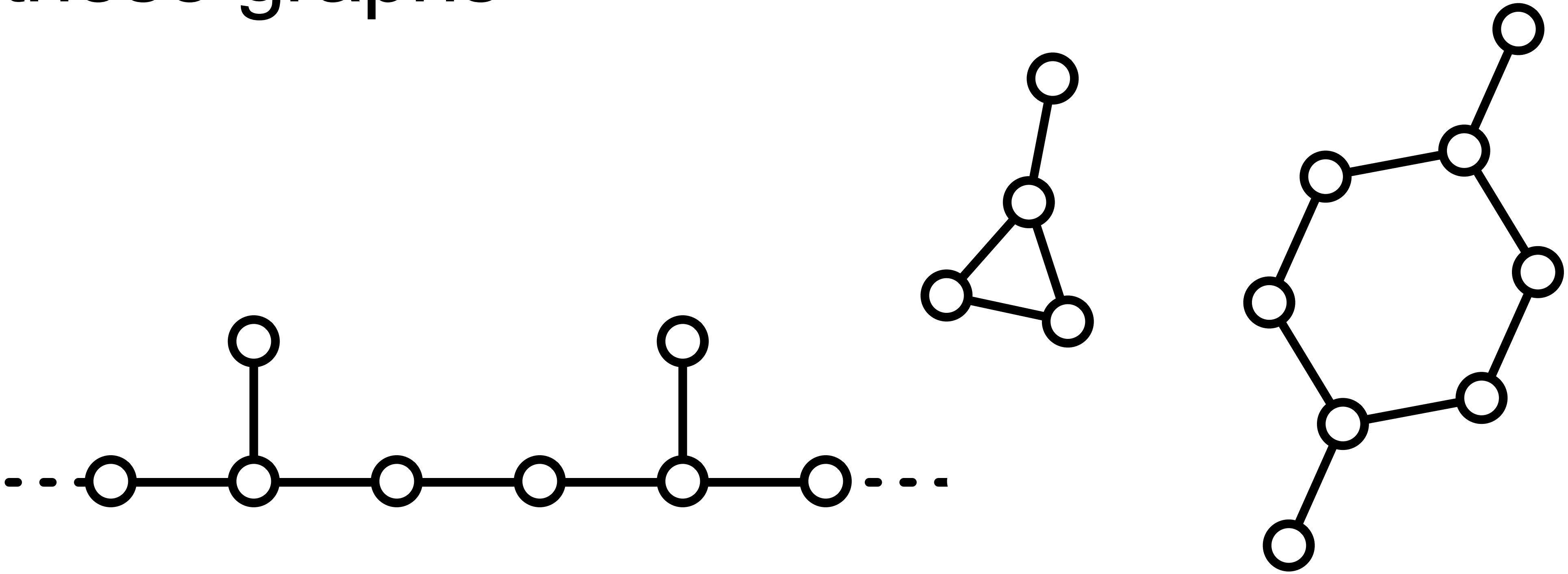
- Example: graph coloring
- Input graph: 
- Impossible to solve!

Breaking symmetry

- Input graph: 
- ***Proof:***
  - same state before round t
  - same outgoing messages
  - same incoming messages
  - same state after round t

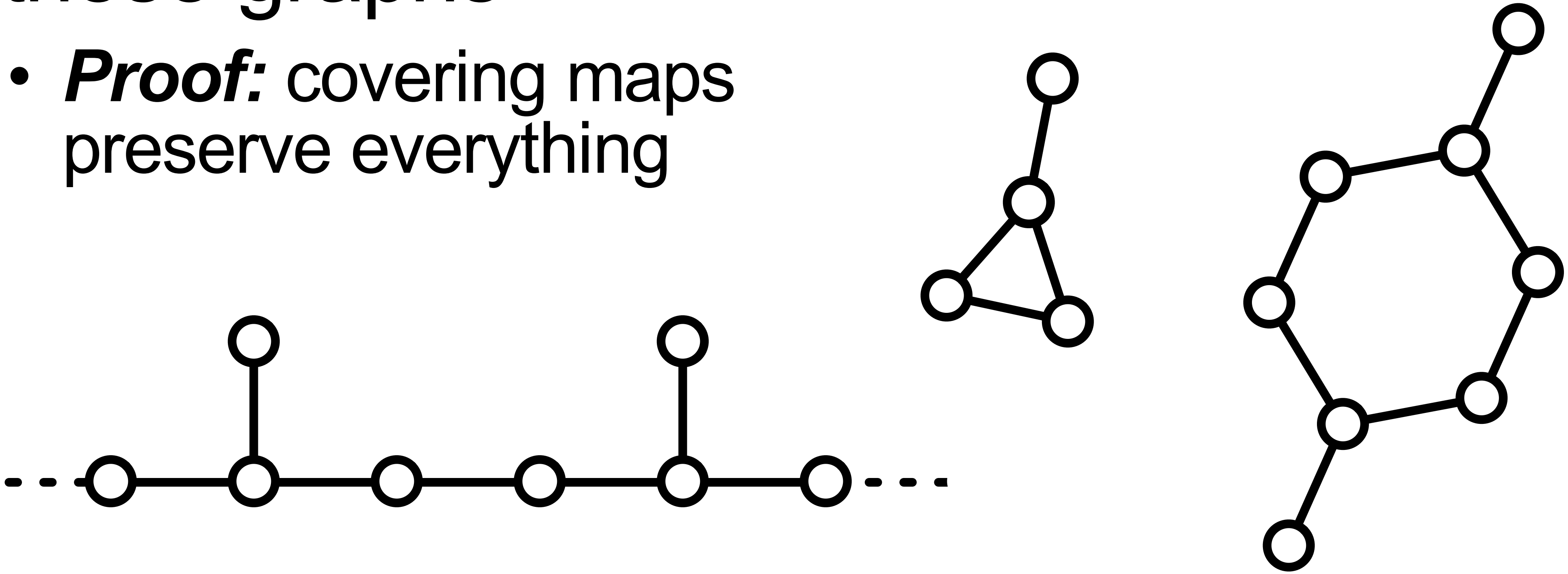
Detecting cycles

- Not possible to tell the difference between these graphs



Detecting cycles

- Not possible to tell the difference between these graphs
  - ***Proof:*** covering maps preserve everything



Weak models

- Lots of different models of distributed computing
  - **“VV”, “MV”, “SV”, “VB”, “MB”, “SB” …**
- Key questions about each model:
  - which problems can be solved ***at all***?
  - which problems can be solved ***in constant time***?

Logical characterizations

Weak models & modal logic

- Natural 1:1 correspondence between:
  - constant-time distributed ***algorithms*** set–broadcast model
  - ***formulas*** in basic modal logic
- Both ***equally expressive***: can “solve” the same set of graph problems

Modal logic & computing

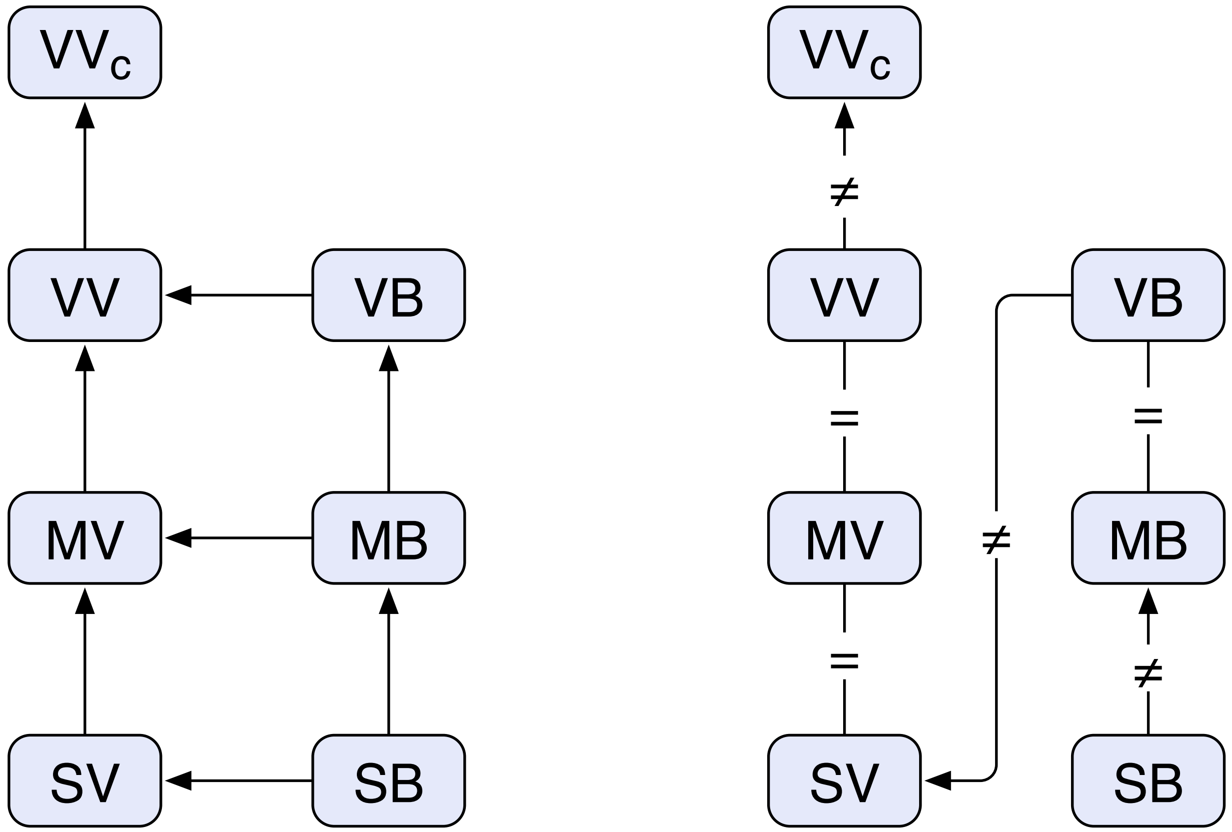
- Textbook approach:
  - possible world ≈ possible **state** of the system
  - accessibility relation ≈ **state transition**
- Our perspective:
  - possible world ≈ **computer**
  - accessibility relation ≈ **communication link**

Modal logic	Distributed algorithms
Kripke model $K = (W, (R_\alpha)_{\alpha \in I}, \tau)$	input graph $G = (V, E)$ port numbering p
states W	nodes V
relations R_α, $\alpha \in I$	edges E and port numbering p
valuation τ proposition symbols $q_1, q_2, \ldots$	node degrees (initial state)
formula φ	algorithm $\mathcal{A}$
formula φ is true in state v	algorithm $\mathcal{A}$ outputs 1 in node v
modal depth of φ	running time of $\mathcal{A}$

Technology transfer

Using tools from logic to prove results on distributed computing

e.g. **bisimulation**



What has happened since 2010?

Beyond constant time

- Easy: running time ≈ operator depth
- Much more challenging to capture: non-constant running time

Beyond constant time

- Promising approach: ***fixed-point logic***
  - e.g. **modal μ-calculus**
  - **Antti Kuusisto** (CSL 2013)
  - **Fabian Reiter** (ICALP 2017)

Nondeterminism & alternation

- ***Stronger*** models of distributed computing
  - cf. nondeterministic & alternating Turing machines
  - cf. class NP & polynomial hierarchy
- Logical characterizations:
  - “alternating local distributed automata”
    ≈ **monadic second-order logic**
  - **Fabian Reiter** (LICS 2015)

Nondeterminism & alternation

- Active research topic: ***distributed decision***
  - yes-instance: **all nodes say "yes"**
  - no-instance: **at least one node says "no"**
- E.g.: ***$O(\log n)$ bits per node*** per quantifier
  - **Göös, S.** (PODC 2011)
  - **Feuilloley, Fraigniaud, Hirvonen** (ICALP 2016)

What is happening right now?

Structural complexity theory

- ***Centralized computing:***
  - **time hierarchy theorem**
  - more time $\rightarrow$ can solve more problems
- ***Distributed computing:***
  - **gap results**
  - $o(\log n)$ rounds $\approx$ as good as $O(\log^* n)$ rounds

Structural complexity theory

- Key idea that has enabled lots of progress: ***identify the right family of problems***
  - do not try to prove something about "**all graph problems**"
  - focus on "**LCL problems**" (locally checkable labeling)
  - distributed analogue of class NP: solutions are easy to verify, but may be hard to find

Structural complexity theory

- *Lots of progress:*
  - **Brandt et al.** (STOC 2016)
  - **Chang et al.** (FOCS 2016)
  - **Ghaffari & Su** (SODA 2017)
  - **Brandt et al.** (PODC 2017)
  - **Chang & Pettie** (FOCS 2017)
  - **Balliu et al.** (STOC 2018) …

Structural complexity theory

- One of the current obstacles: we seem to be still ***lacking the right definitions***
  - example: **LCLs** work well for graphs of maximum degree $O(1)$, but how to **generalize** beyond that?
- Could we try to replace the current algorithmic or graph-theoretic definitions with ***logical characterizations***?

Thanks!
Happy Birthday!

Helsinki · Finland
20-24 August 2018

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