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Causal limits

of distributed quantum computation

Joint work with

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- Fabian Kuhn
- François Le Gall
- Henrik Lievonen
- Augusto Modanese
- Marc-Olivier Renou
- Gustav Schmid …

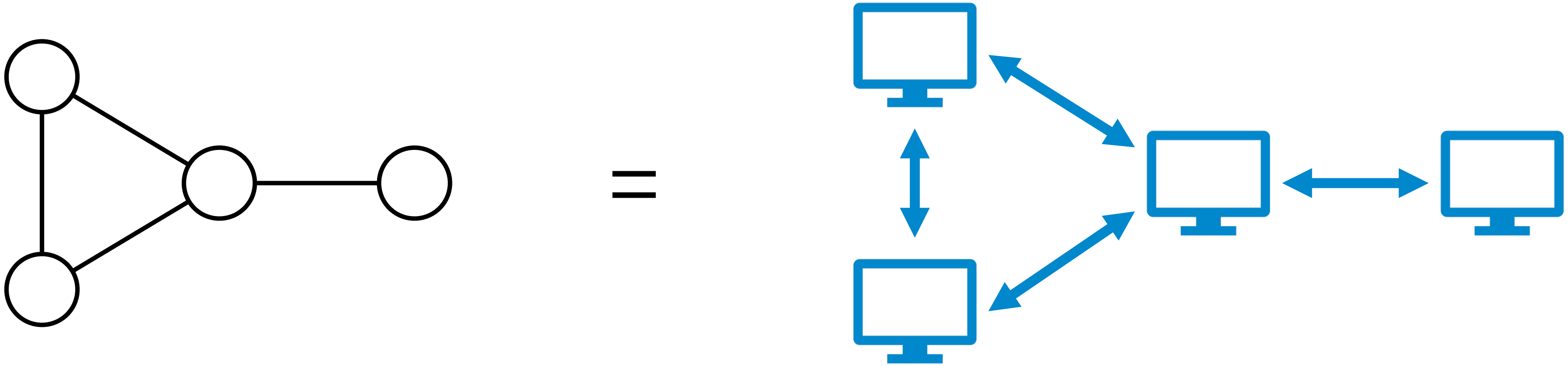
arxiv.org/abs/2307.09444

Quick recap:

Classical distributed algorithms

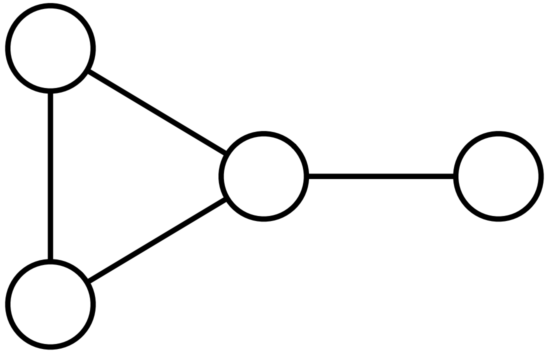
Distributed algorithms

- Graph = communication network
  - node = computer
  - edge = communication link



Distributed algorithms

- **Initially:** each node only aware of itself
- **Goal:** each announces its local output
  - e.g. graph coloring: “my color is 5”



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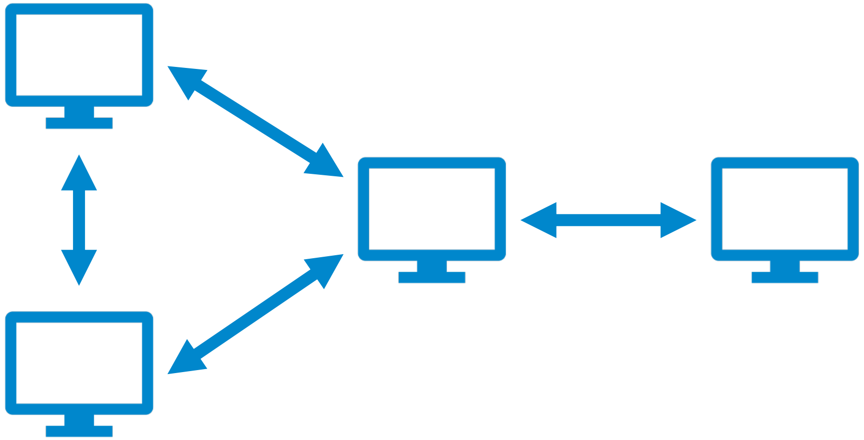
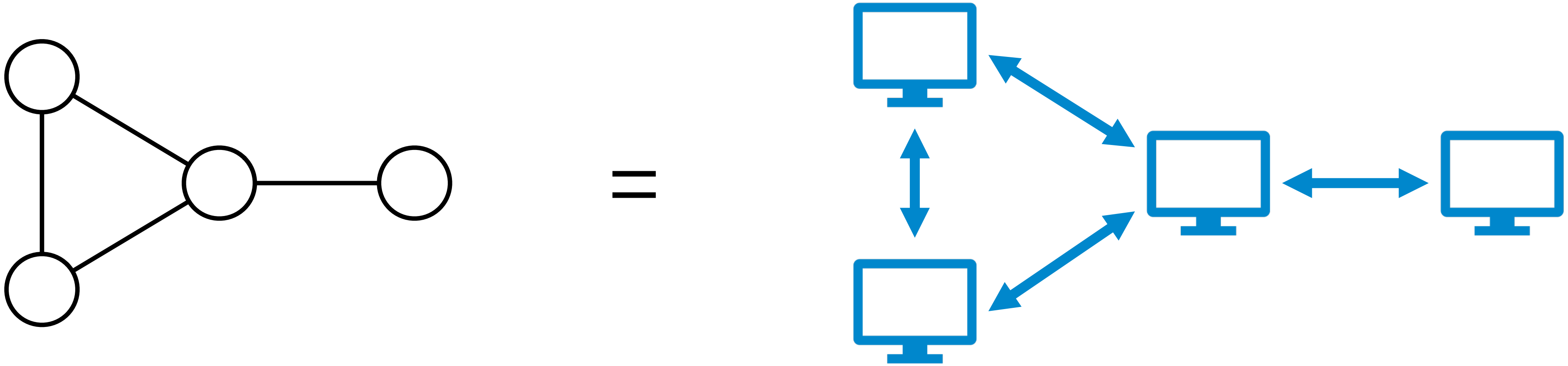
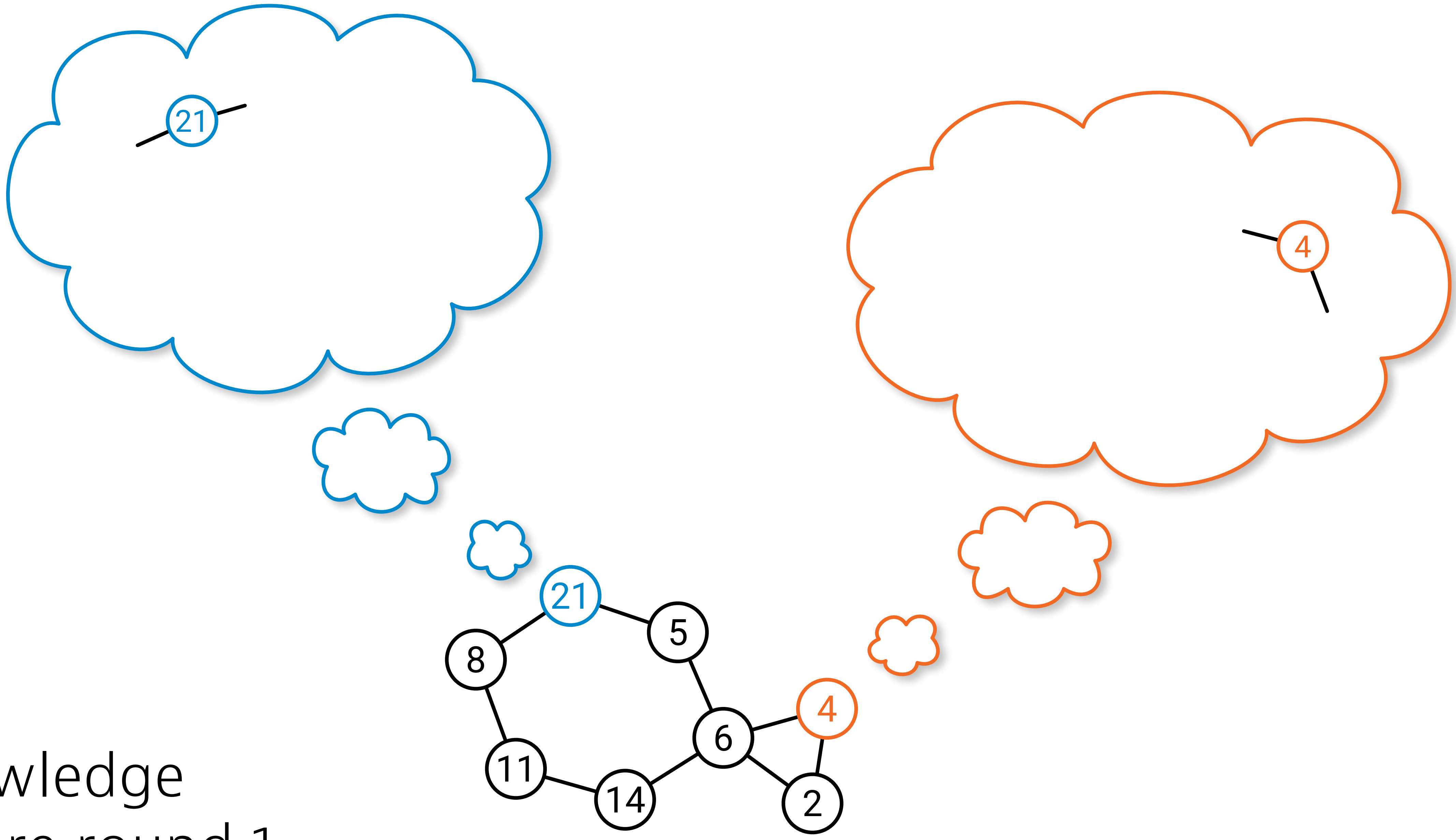


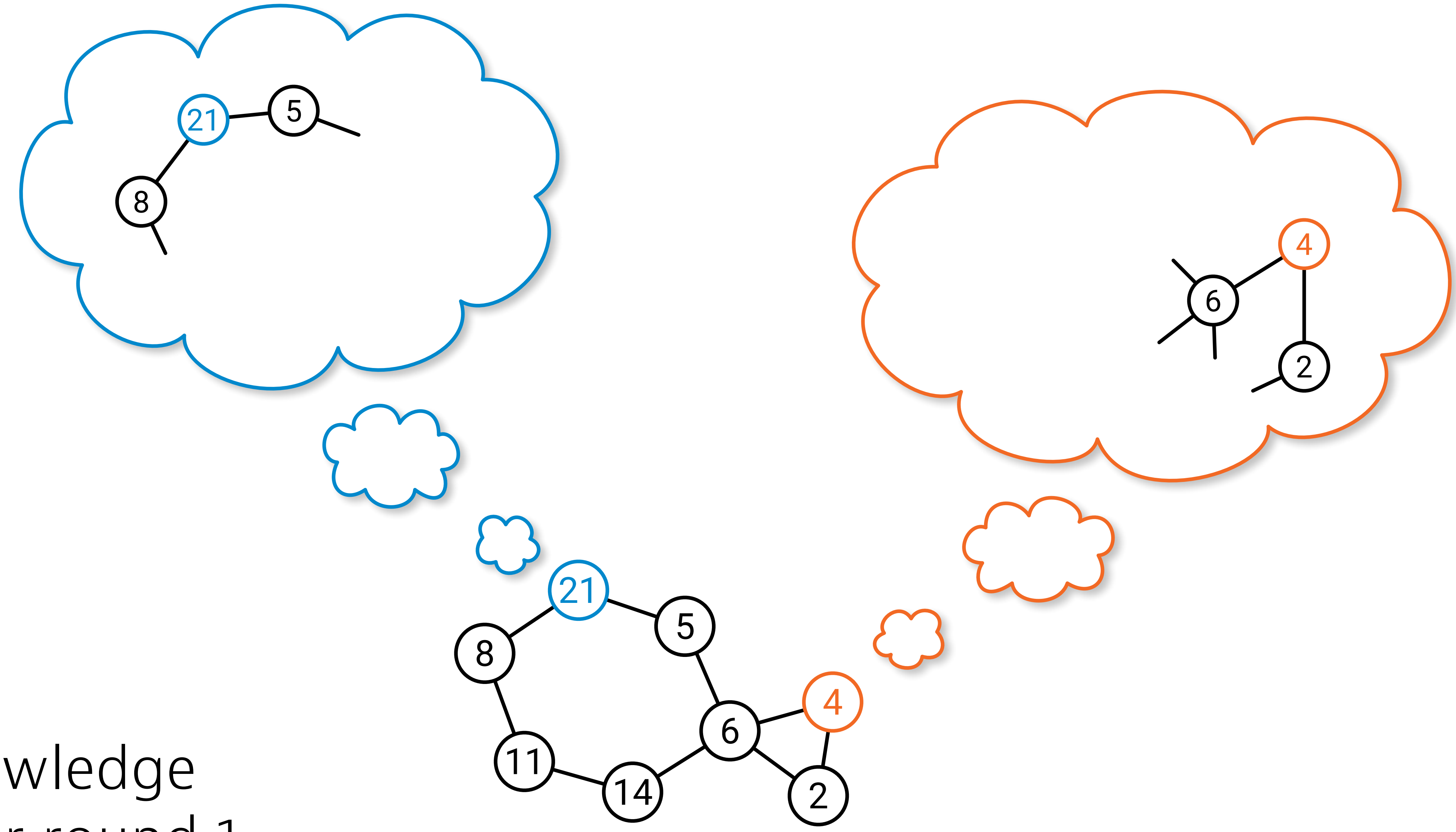
Distributed algorithms

- **Running time = number of rounds** until all nodes have stopped

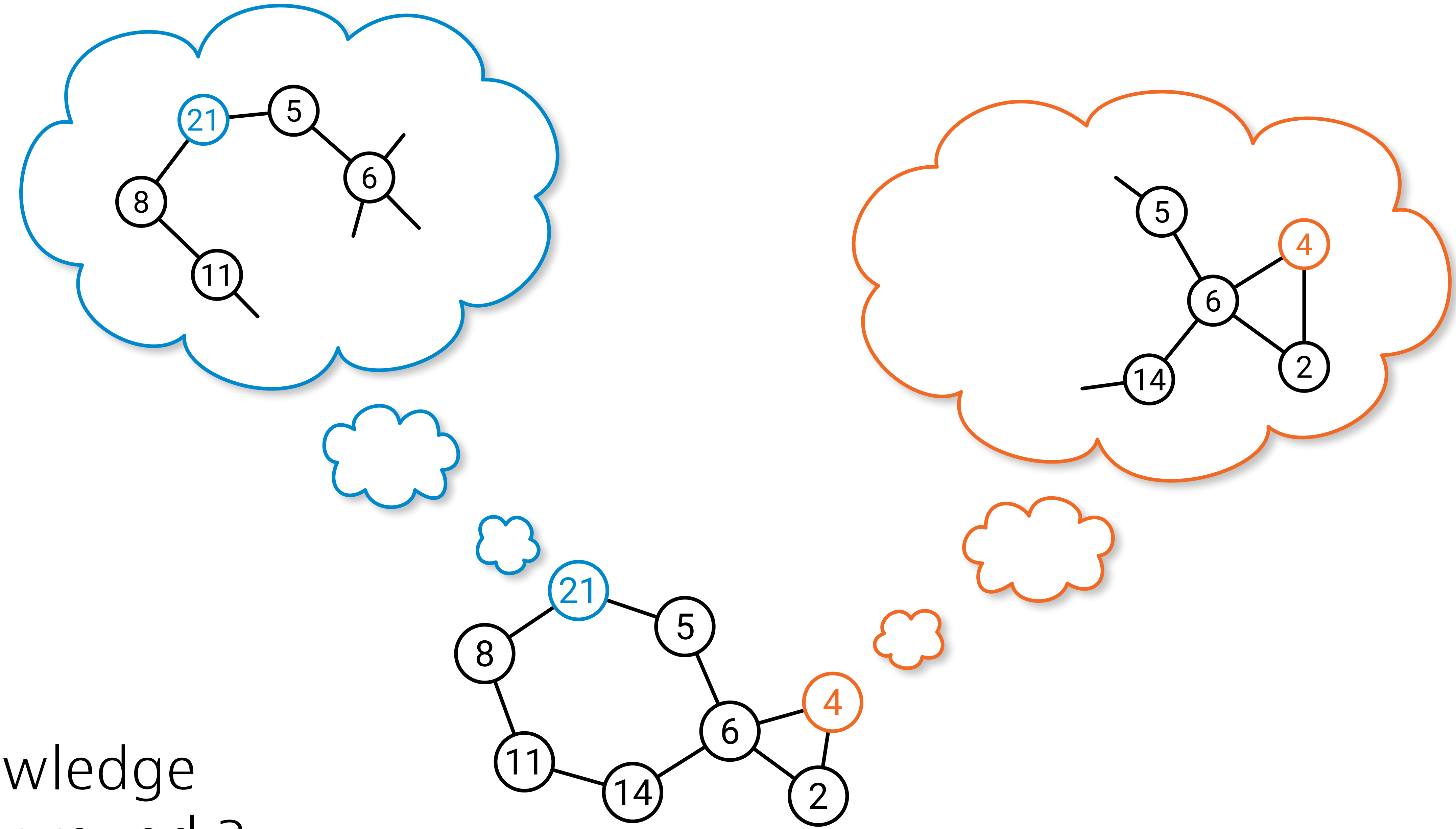




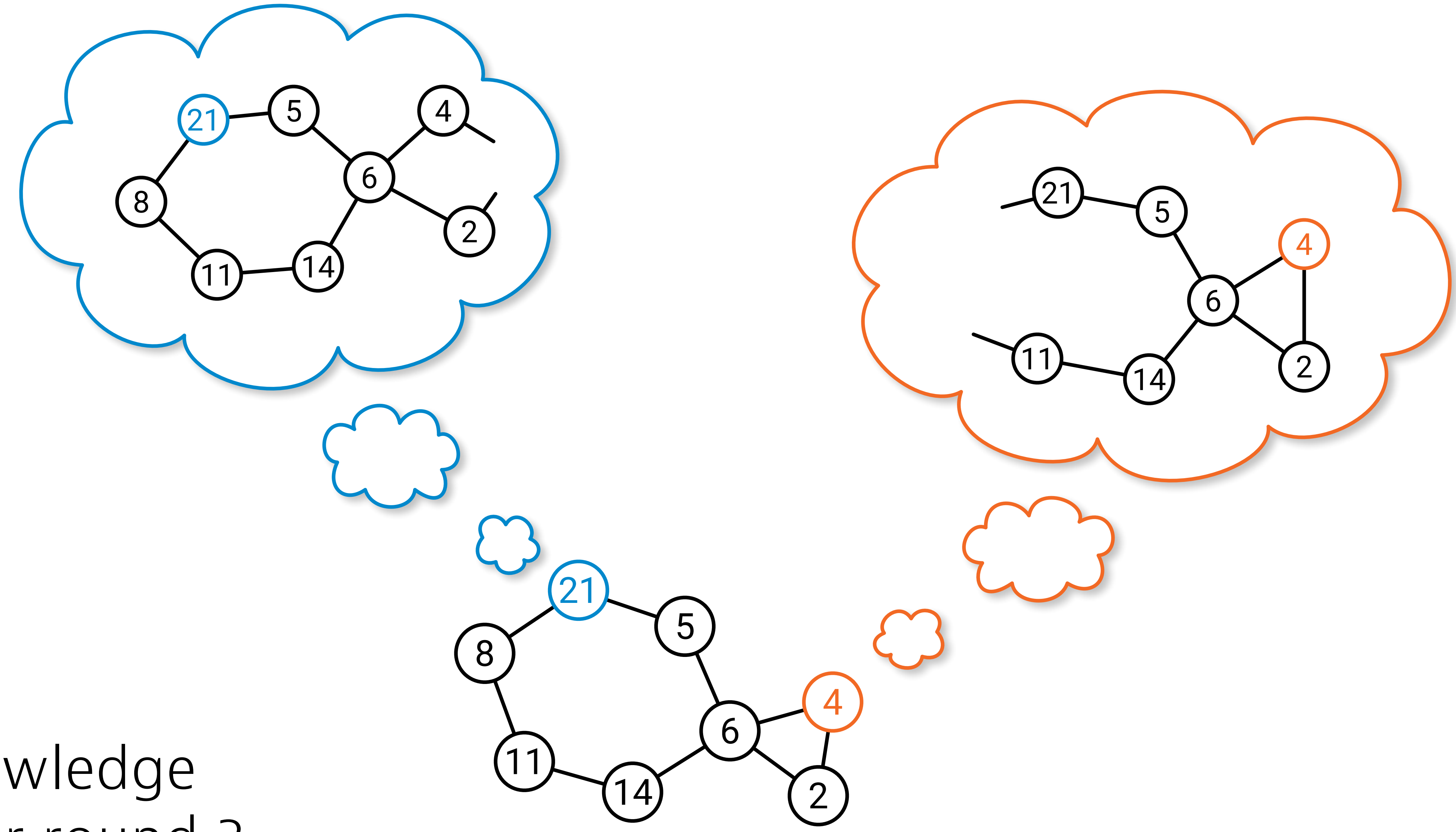
Knowledge
before round 1



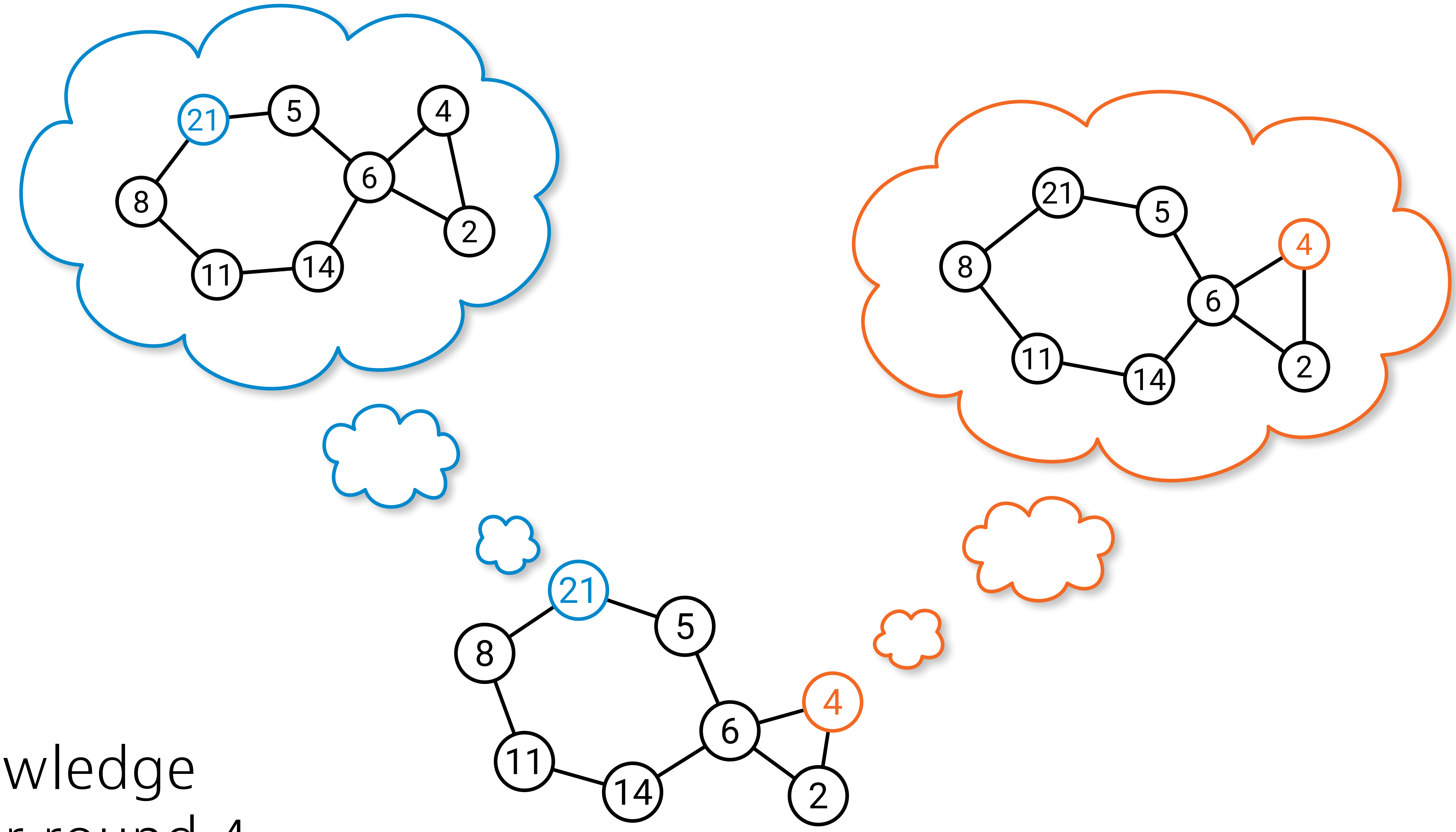
Knowledge
after round 1



Knowledge
after round 2



Knowledge
after round 3



Knowledge
after round 4

number of communication rounds

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how far do you need to see

What about quantum?

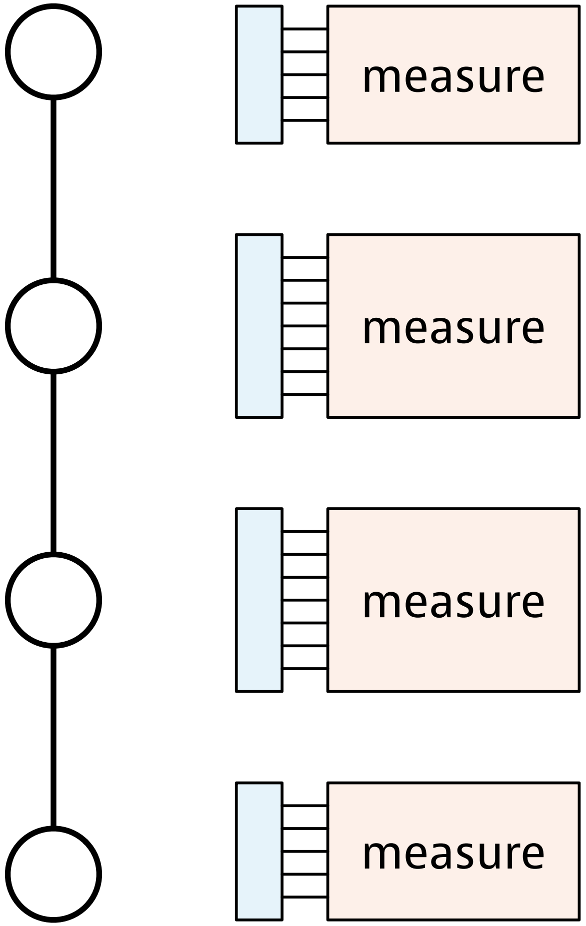
Distributed algorithms

Classical

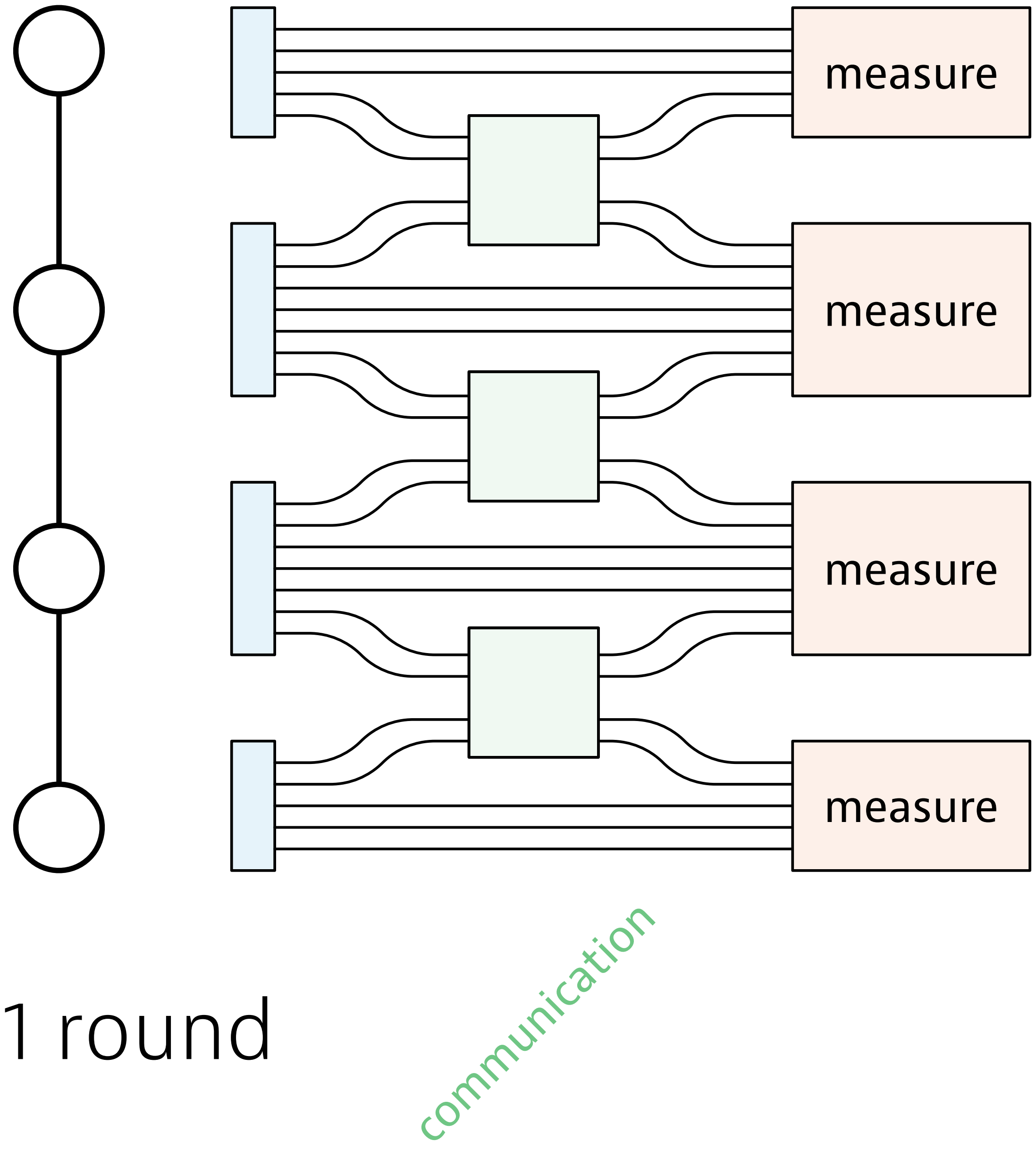
- node = classical computer
- edge = classical communication channel

Quantum

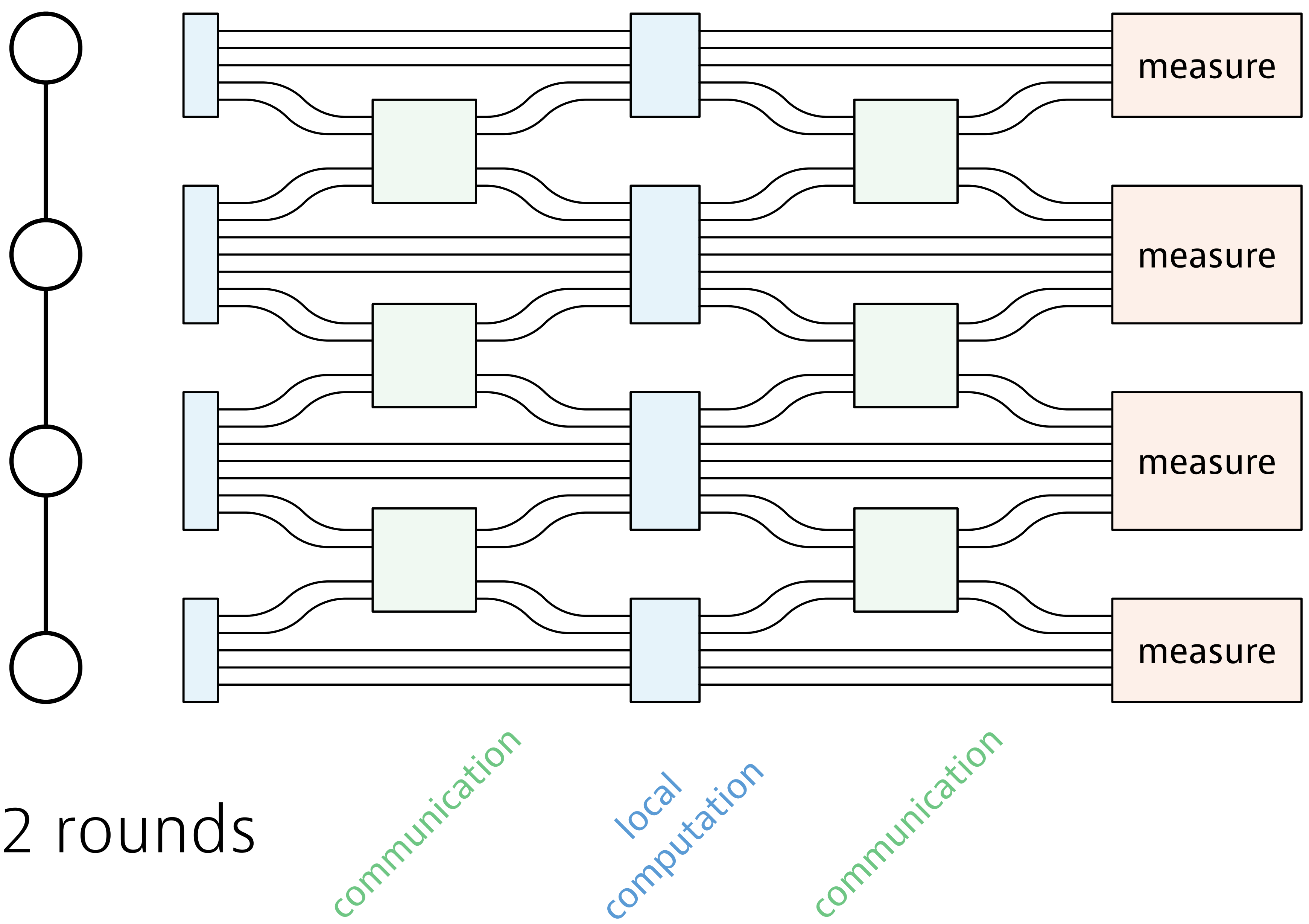
- node = quantum computer
- edge = quantum communication channel



0 rounds

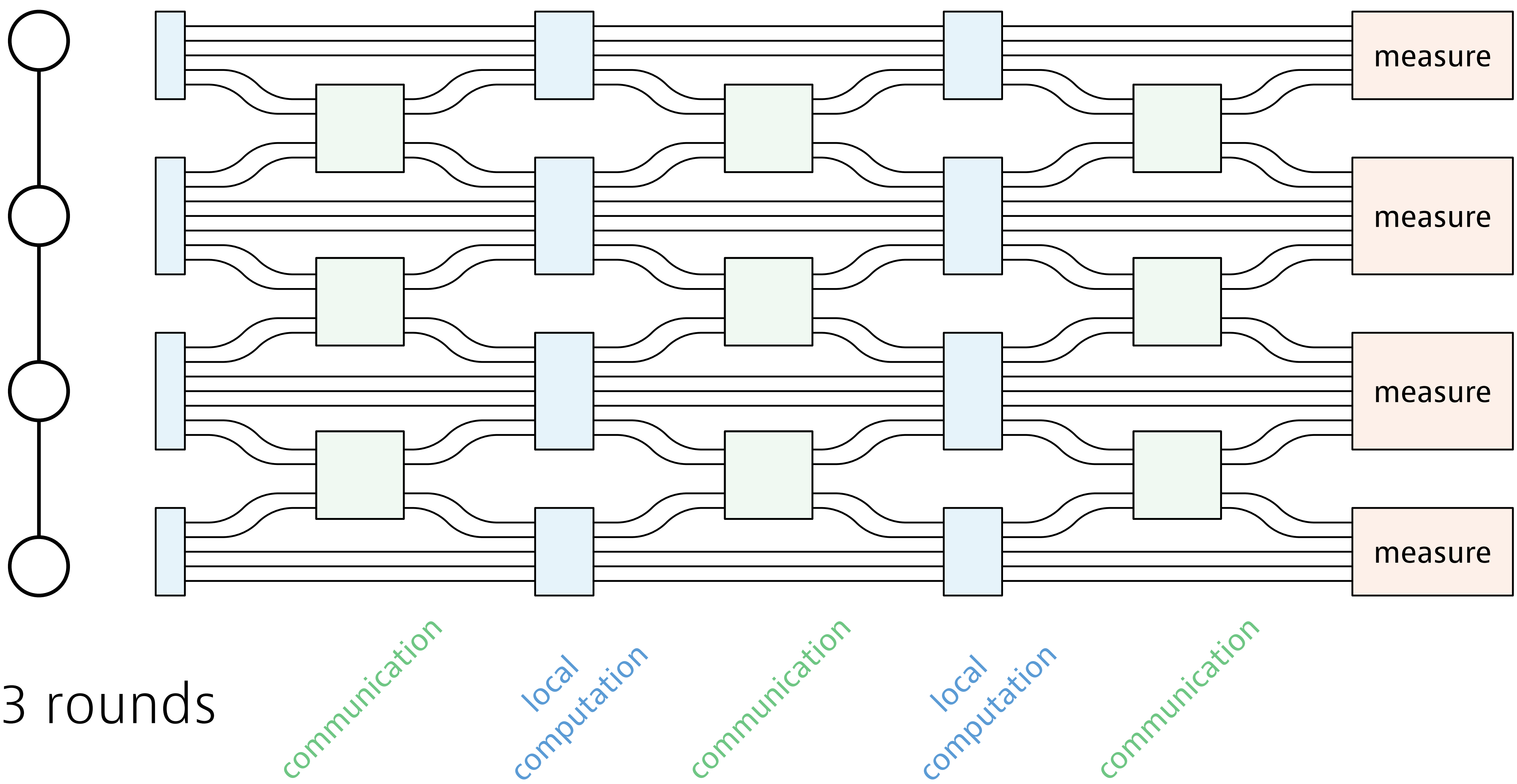


1 round



measure
measure
measure
measure
communication
local
computation
communication

2 rounds


measure
measure
measure
measure
communication
local computation
communication
local computation
communication

3 rounds

Does it help?

Quantum advantage

- Is there any graph problem for which we can show **distributed quantum advantage**?
- Yes!
  - $O(1)$-round quantum algorithm
  - no $o(n)$-round classical algorithm

Le Gall, Nishimura, Rosmani 2019

Quantum advantage

- Is there any **graph problem that someone actually cares about** for which we can show distributed quantum advantage?
- Nobody knows!

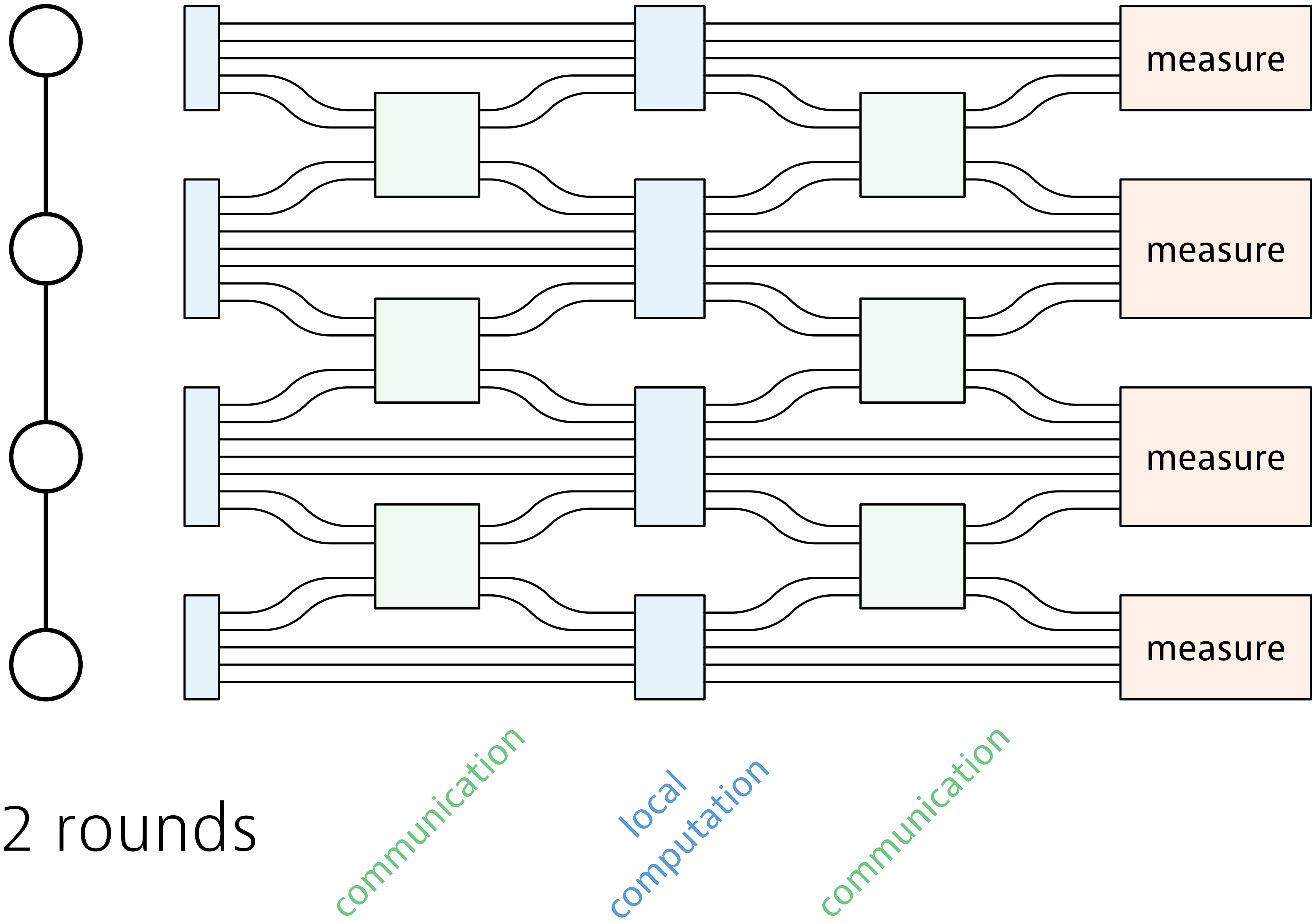
Quantum advantage

- It is hard to characterize the exact limits of distributed quantum computation
  - *and you need to understand quantum things…*
- But for many graph problems we can use **causality** to show that there is **no** quantum advantage!
  - *without knowing anything about quantum things!*

Causality

Not faster than light

- No physical thing can allow faster-than-light communication
- Not even your quantum distributed algorithm


measure
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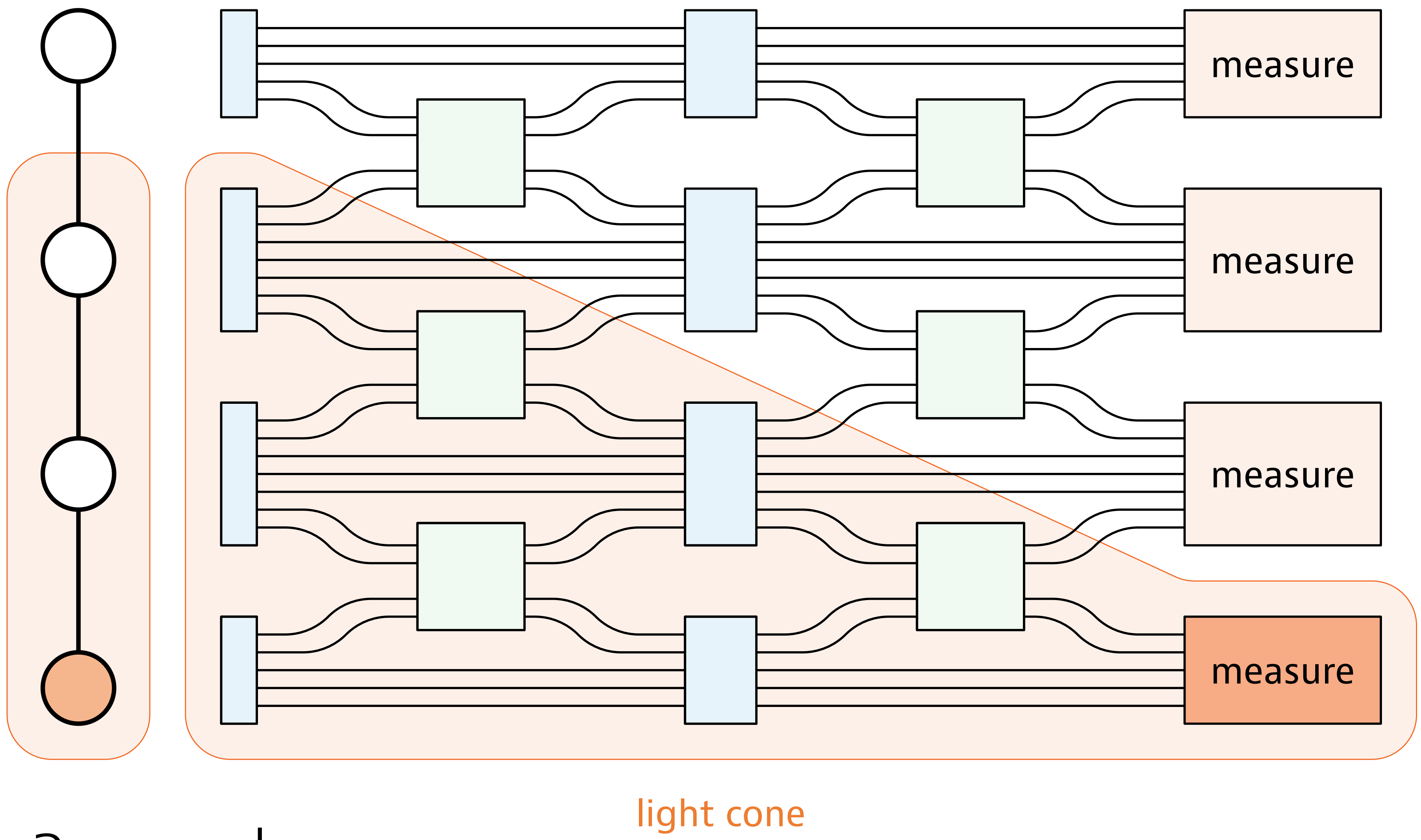
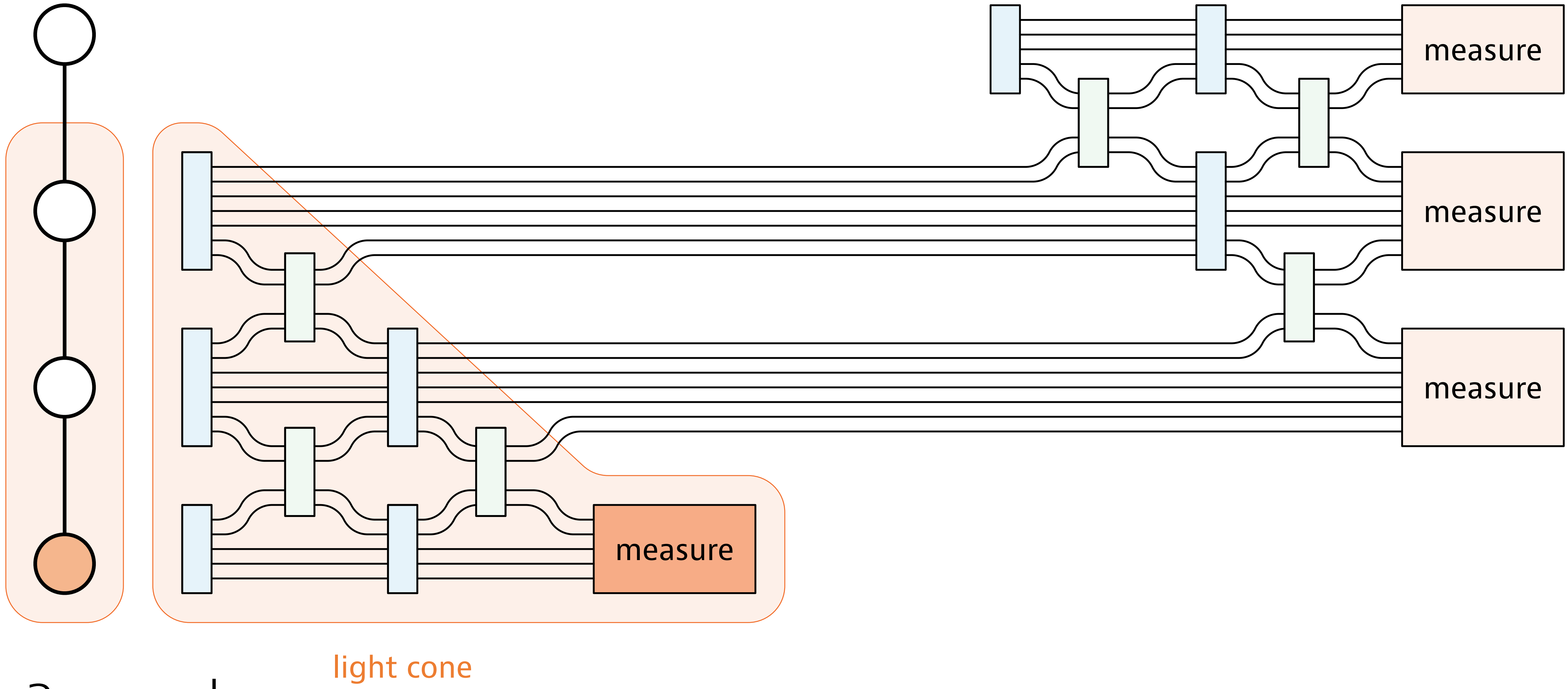
2 rounds



2 rounds



2 rounds

Non-signaling model

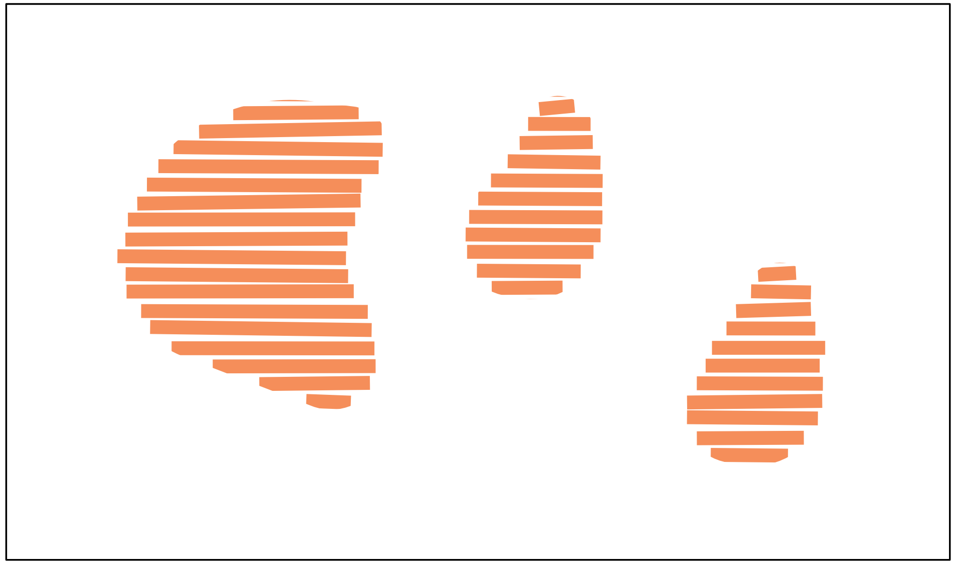
- Key idea: **define** a model so that it can do **anything** except violating causality

Non-signaling model

Definition (*non-signaling distribution*):

- fix any **set of nodes *X*** …

Gavoille, Kosowski, Markiewicz 2009
Arfaoui, Fraigniaud 2014



Non-signaling model

Definition (*non-signaling distribution*):

- fix any **set of nodes *X***
- changes in the input **more than *T* hops away** from ***X*** do not influence the output distribution of ***X***

Gavoille, Kosowski, Markiewicz 2009
Arfaoui, Fraigniaud 2014

Three models

Classical probability theory

Classical (randomized) distributed algorithms

↓

Quantum distributed algorithms

Weird quantum things

↓

Non-signaling "algorithms"

Classical probability theory

Example: graph coloring

- Any quantum advantage?
- **No!** (up to polylog factors)

Example: graph coloring

- Any quantum advantage?
- **No!** (up to polylog factors)
- Example: **3-coloring bipartite graphs**
  - $\tilde{O}(n^{1/2})$ — classical distributed algorithm
  - $o(n^{1/2})$ — impossible for non-signaling

Example: graph coloring

- Any quantum advantage?
- **No!** (up to polylog factors)
- Example: **4-coloring bipartite** graphs
  - $\tilde{O}(n^{1/3})$ — classical distributed algorithm
  - $o(n^{1/3})$ — impossible for non-signaling

Example: graph coloring

- Any quantum advantage?
- **No!** (up to polylog factors)
- Example: **25-coloring 7-colorable** graphs
  - $\tilde{O}(n^{1/4})$ — classical distributed algorithm
  - $o(n^{1/4})$ — impossible for non-signaling

Summary

- **Causality**
  - no faster-than-light communication
  - “non-signaling model”
- **Tight lower bounds for distributed quantum algorithms**
  - without touching weird quantum things

An open question

- **3-coloring cycles:**
  - classical: $O(\log^* n)$
  - quantum: **???**
  - non-signaling: $O(1)$

Linial 1992
Holroyd, Liggett 2016
Holroyd, Hutchcroft, Levy 2018