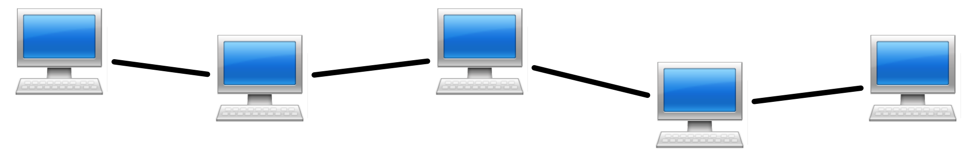
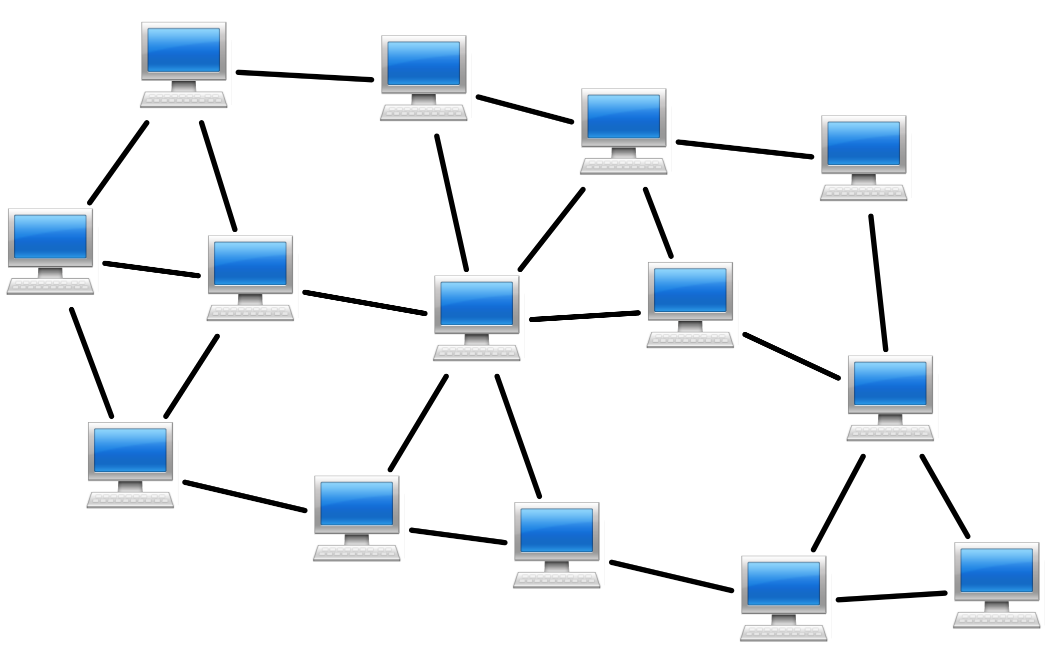
- **Weeks 1–2: informal introduction**
  - network = path 
- **Week 3: graph theory**
- **Weeks 4–7: models of computing**
  - what can be computed (efficiently)?
- **Weeks 8–11: lower bounds**
  - what cannot be computed (efficiently)?
- **Week 12: recap**

Week 12

– Conclusions

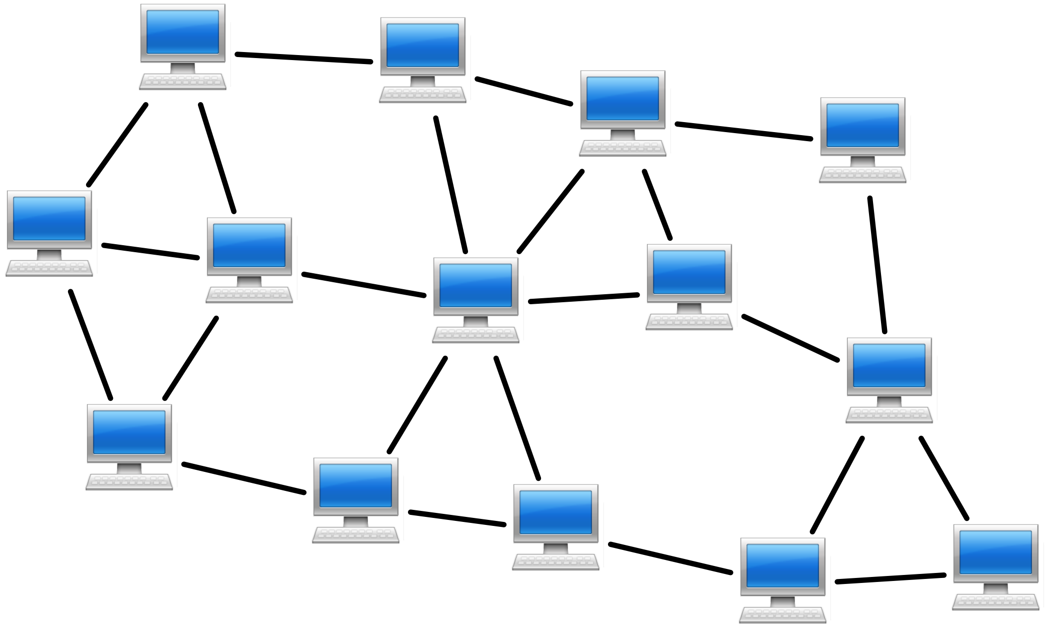
Recap:
Distributed algorithms

Algorithms for computer networks



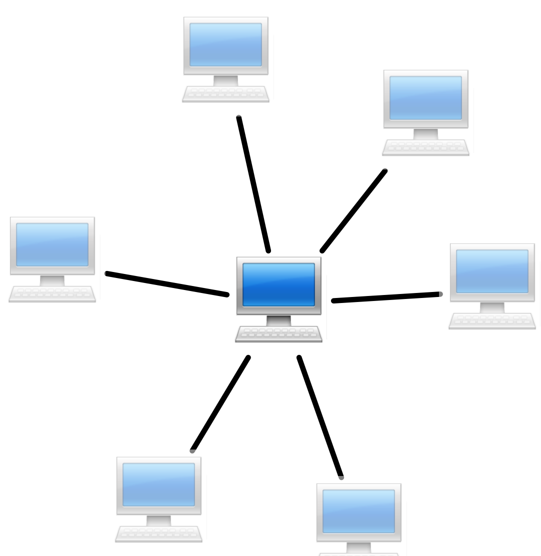
Recap:
Distributed algorithms

Identical computers in an unknown network, all running the same algorithm



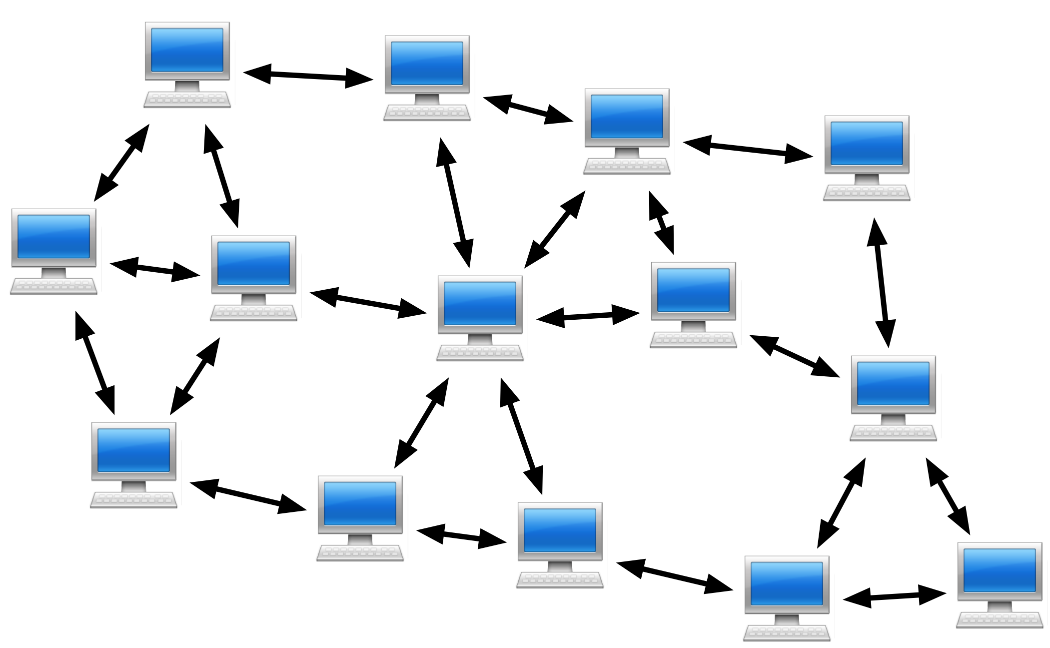
Recap:
Distributed algorithms

Initially each computer only aware of its immediate neighbourhood



Recap:
Distributed algorithms

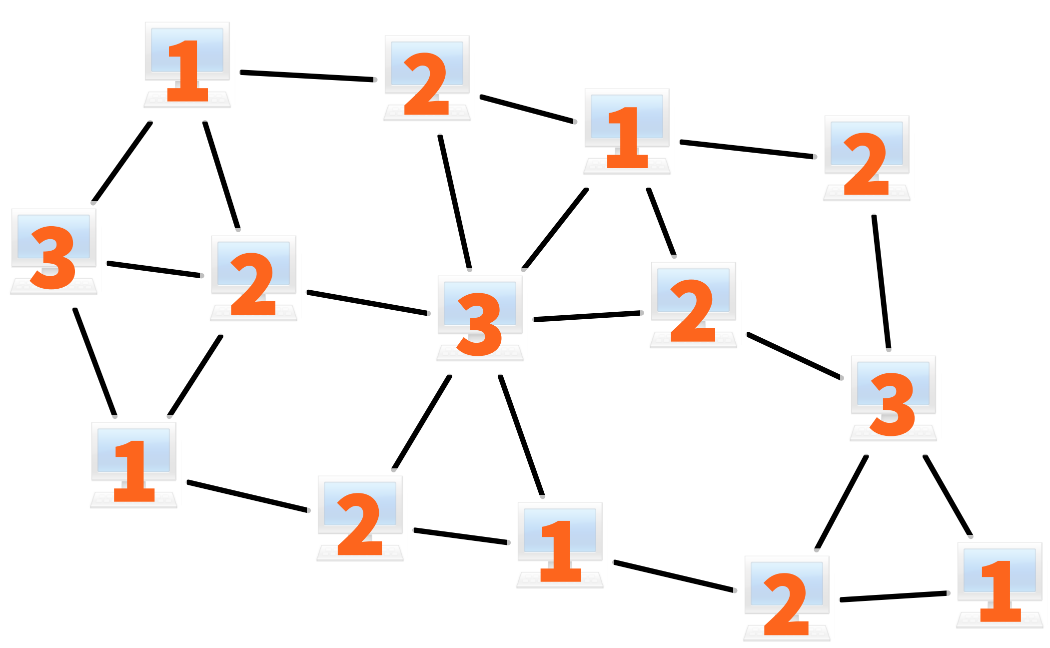
Nodes can exchange messages with their neighbours to learn more…



Recap:

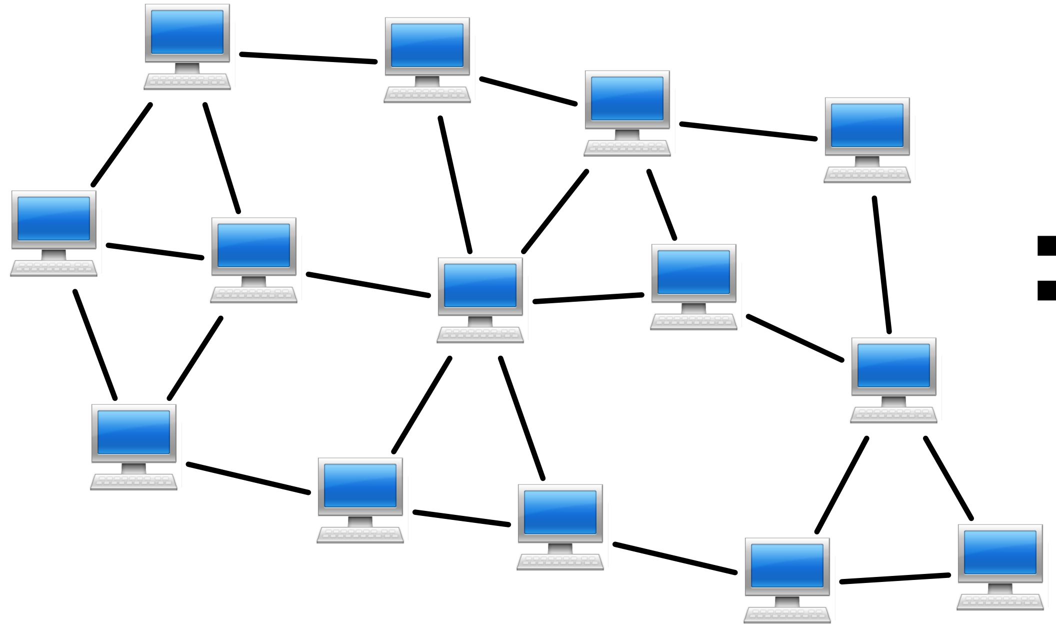
Distributed algorithms

Finally, each computer has to stop and produce its own local output



Recap:
Distributed algorithms

Focus on graph problems:
network topology = input graph



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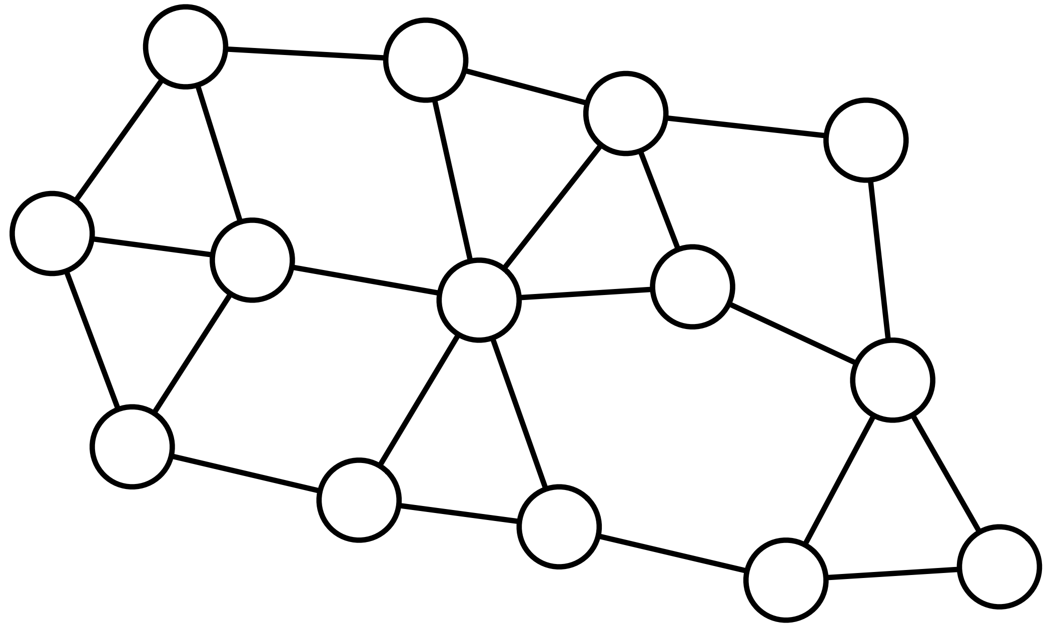
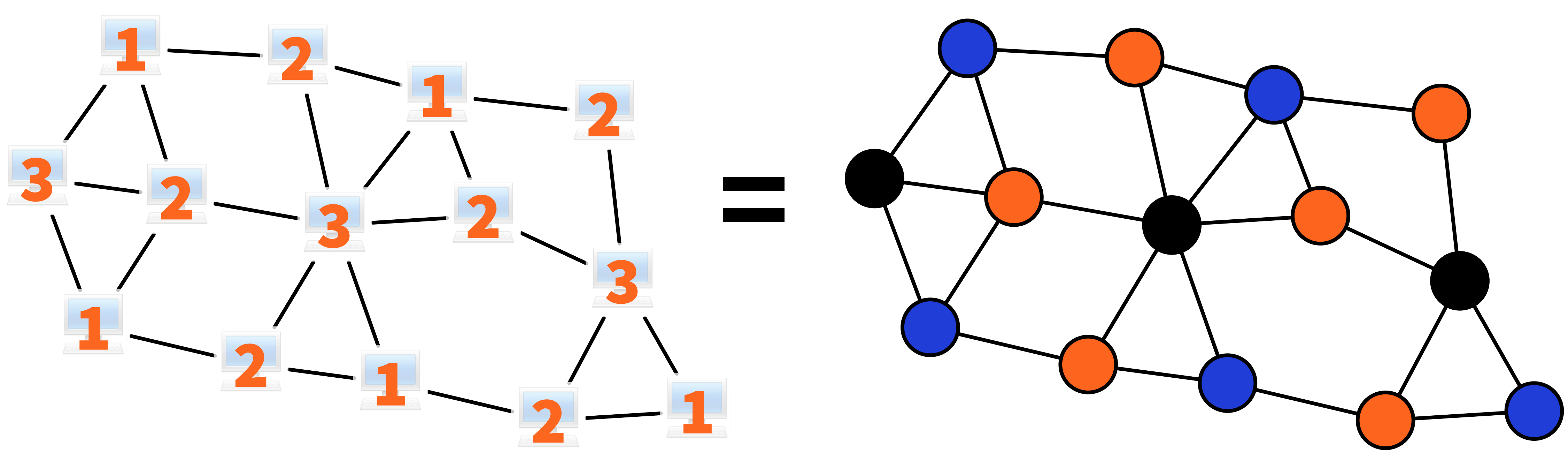


Recap:
Distributed algorithms

Focus on graph problems:
local outputs = solution (here: graph colouring)



Recap:
Distributed algorithms

Typical research question:

"How fast can we solve graph problem X?"

Time = number of communication rounds

What have we learned?

- **Dealing with *unknown systems***
- **Dealing with *partial information***
- **Dealing with *parallelism***
- **Applications beyond distributed computing:** fault tolerance, online, streaming, multicore…

Learning objectives

- Models
- Algorithms
- Lower bounds
- Graph theory

Objective 1:
Models of computing

- **Precisely what is a *"distributed algorithm"***
- **In each of these models:**
  - PN, LOCAL, CONGEST
  - deterministic, randomised

Objective 2: **Algorithms**

- **Colouring paths:** LOCAL, $O(\log^* n)$
- **Colouring graphs:** LOCAL, $O(\log n)$ w.h.p.
- **Gather everything:** LOCAL, $O(\mathrm{diam}(G))$
- **Bipartite maximal matching:** PN, $O(\Delta)$
- **All-pairs shortest paths:** CONGEST, $O(n)$

Week 1

Algorithm P3CBit: **Fast colour reduction**

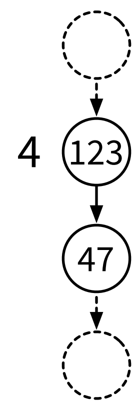
$\boldsymbol{c_0} = \mathbf{123} = \mathbf{01111011_2}$ (my colour)

$\boldsymbol{c_1} = \mathbf{47} = \mathbf{00101111_2}$ (successor's colour)

$\boldsymbol{i} = \mathbf{2}$ (bits numbered 0, 1, 2, … from right)

$\boldsymbol{b} = \mathbf{0}$ (in my colour bit number i was 0)

$\boldsymbol{c} = \mathbf{2 \cdot 2 + 0 = 4}$ (my new colour)



$k = 8$, reducing from $2^8 = 256$ to $2 \cdot 8 = 16$ colours

Algorithm P3CBit:
Fas

c_0 = 12
c_1 = 4
i = 2 (
b = 0 (

c = 2·2

k

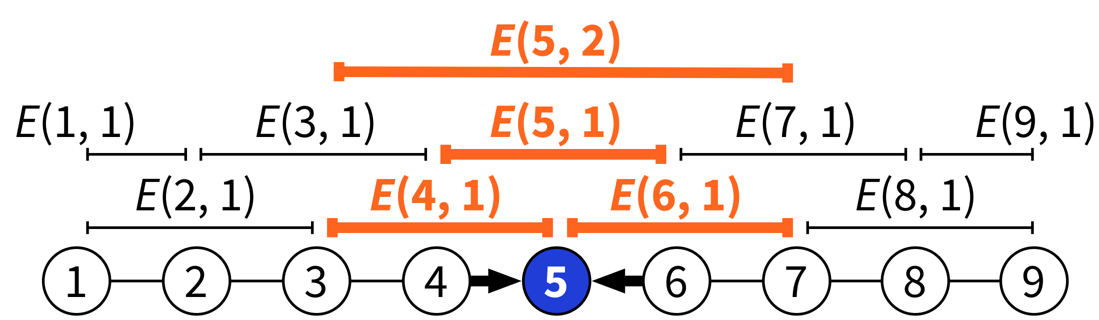
Algorithm idea 3

- **Colour palette: {1, 2, ..., Δ + 1}**
- **Active with probability 1/2**
- **If *active*, pick a random *free* colour**
  - not used by any neighbour that has stopped
- **Try again if conflicts...**

Week 5

Gathering everything

- **Given $E(v, r)$, we can learn $E(v, r + 1)$ in 1 round**
  - send $E(v, r)$ to all neighbours, take union

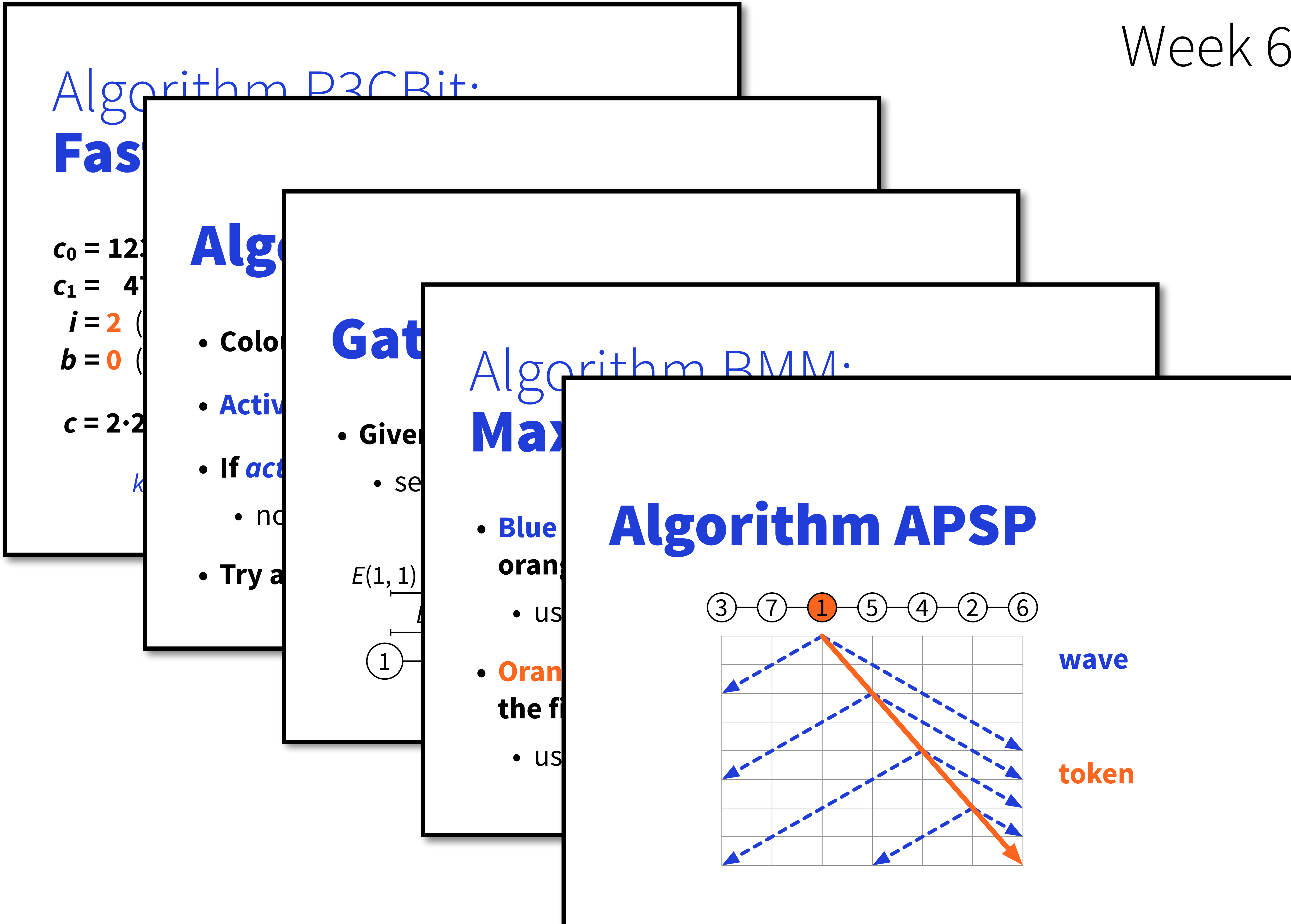


Week 4

Algorithm BMM: **Maximal matching**

- **Blue nodes send proposals to their orange neighbours one by one**
  - using port numbers
- **Orange nodes accept the first proposal that they get**
  - using port numbers to break ties

Week 6



Objective 2: **Algorithms**

- **Reductions!**
- **Graph colouring is a very useful subroutine**

Objective 3:
Lower bounds

- **Covering maps:**
  what cannot be solved at all in PN model
- **Local neighbourhoods:**
  what cannot be solved fast in any model
- **Ramsey’s theorem:**
  what cannot be solved in $O(1)$ time

Objective 4:
Graph theory

- **Basic definitions**
- **Connections between graph problems**
  - e.g. maximal matching → small vertex covers
- **Ramsey's theorem**
  - at least for $c = 2$, $k = 2$

What else is studied in distributed computing?

- Fault-tolerance
- Asynchrony
- Shared memory
- Physical models
- Robot navigation
- Nondeterminism
- Complexity measures
- High-performance computing
- Practical aspects of networking …

What next?

- **ICS-E4020 Programming Parallel Computers**
  - 5th period, 5 credits, intensive course
  - programming modern parallel computers: multicore, GPU, memory hierarchies …
  - hands-on programming exercises
  - main goal: make it as fast as you can!

What next?

- **Just ask if you want to do more!**
  - master’s thesis topics?
  - summer internships?
  - doctoral studies?

Practicalities

- **2nd mid-term exam: 10 December**
  - remember to register on time!
- **Course feedback: deadline 17 December**
  - 1 extra point in grading

What to expect in the exam?

- See the learning objectives!
- Do not think that you can safely forget what we learned during the 1st period!
- Expect both algorithm design and lower bound proofs

Examples of old exam problems

- Prove: no deterministic PN-algorithm that finds a *minimum* vertex cover in cycle graphs, given a *minimal* vertex cover

Examples of old exam problems

- Prove: no deterministic PN-algorithm that finds a 6-colouring in cycle graphs given a maximal independent set

Examples of old exam problems

- **Counting problem**: all nodes output $|V|$
- Prove: no deterministic PN-algorithm for cycle graphs
- Prove: no $o(n)$-time deterministic LOCAL-algorithm for cycle graphs

Examples of old exam problems

- Prove: no deterministic PN-algorithm for maximal matching in arbitrary graphs

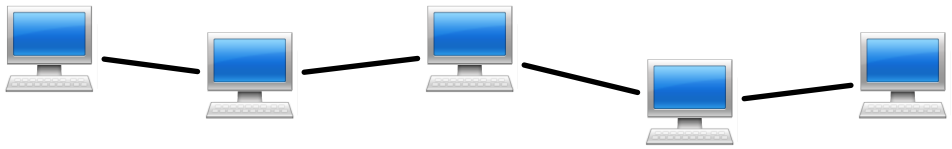
Examples of old exam problems

- Prove: no deterministic $o(n)$-time PN-algorithm for weak 2-colouring in paths of length ≥ 3

Examples of old exam problems

- Give an elementary proof that
  any graph with 6 nodes contains
  a clique with 3 nodes or
  an independent set with 3 nodes

- **Weeks 1–2: informal introduction**
  - network = path



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