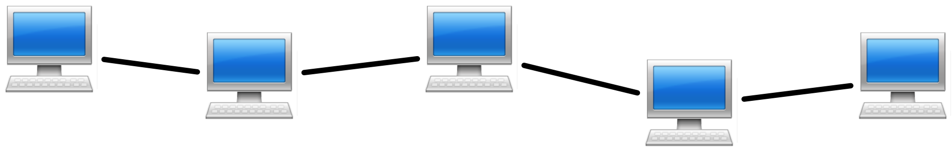
- **Weeks 1–2: informal introduction**
  - network = path 
- **Week 3: graph theory**
- **Weeks 4–7: models of computing**
  - what can be computed (efficiently)?
- **Weeks 8–11: lower bounds**
  - what cannot be computed (efficiently)?
- **Week 12: recap**

Mid-term exams

- **Mid-term exams:**
  - Thursday, 23 October 2014, 9:00am
  - Thursday, 11 December 2014, 9:00am
- **Register on time (*one week before*) in Oodi**

Week 5

– LOCAL model:
unique identifiers

LOCAL model

- **Idea: nodes have unique names**
- **Names arbitrary but fairly short**
- **IPv4 addresses, IPv6 addresses, MAC addresses, IMEI numbers…**

LOCAL model

- **LOCAL model =**
  PN model + unique identifiers
- **Assumption: unique identifiers**
  are given as local inputs

LOCAL model

- **Algorithm has to work correctly for any port numbering and for any unique identifiers**
- **Adversarial setting:**
  - you design algorithms
  - adversary picks graph, port numbering, IDs

LOCAL model

- **Fixed constant c**
- **In a network with n nodes, identifiers are a subset of $\{1, 2, \ldots, n^c\}$**
- **Equivalently: unique identifiers can be encoded with $O(\log n)$ bits**

PN vs. LOCAL

- **PN: few problems can be solved**
- **LOCAL: all problems can be solved** (on connected graphs)

PN vs. LOCAL

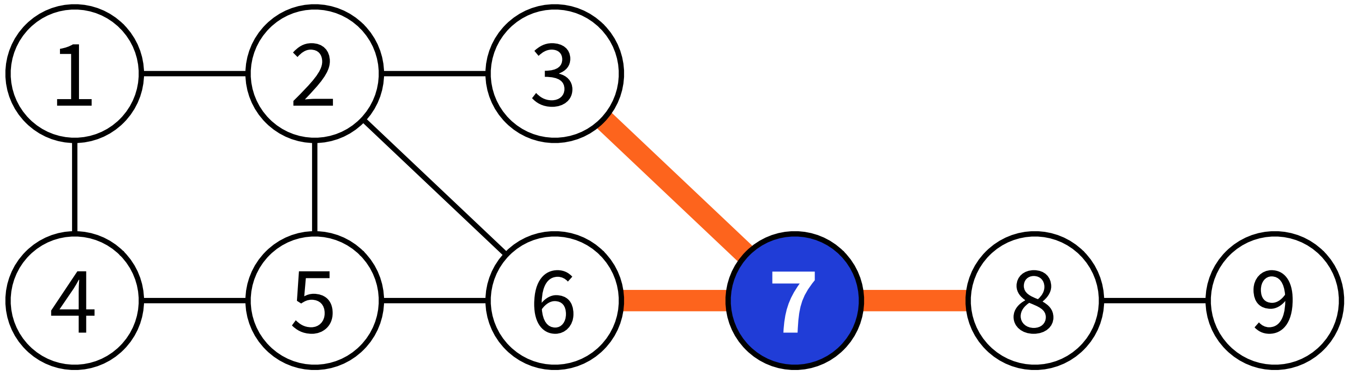
- **PN: “*what can be computed?*”**
- **LOCAL: “*what can be computed efficiently?*”**

Solving everything

- **All nodes learn everything about the graph**
    - $O(\mathrm{diam}(G))$ rounds
- **All nodes solve the problem locally (e.g., by brute force)**
    - 0 rounds

Gathering everything

- $E(v, r)$ = "edges within distance r from v"
  = one endpoint at distance at most $r - 1$ from v



$E(7, 1)$

Gathering everything

- $E(v, r)$ = "edges within distance r from v"
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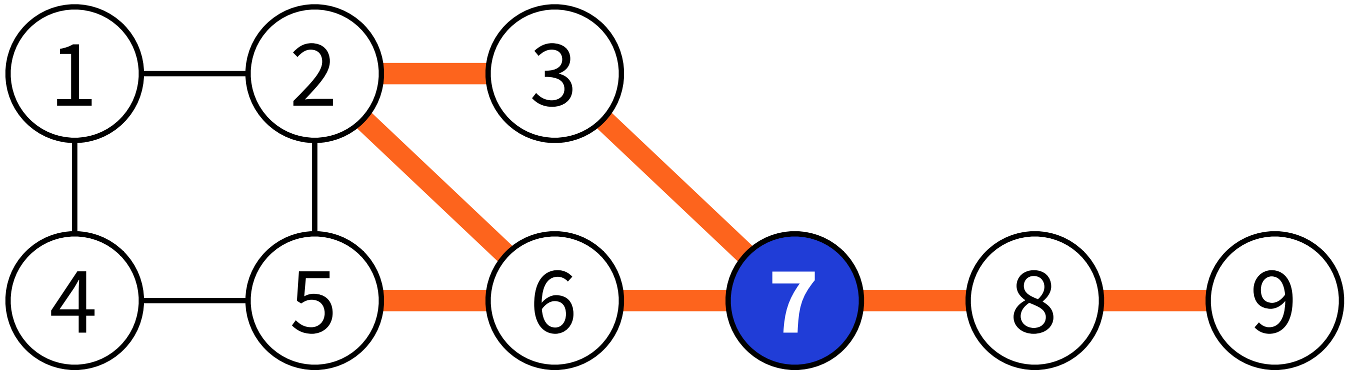

$E(7, 2)$

Gathering everything

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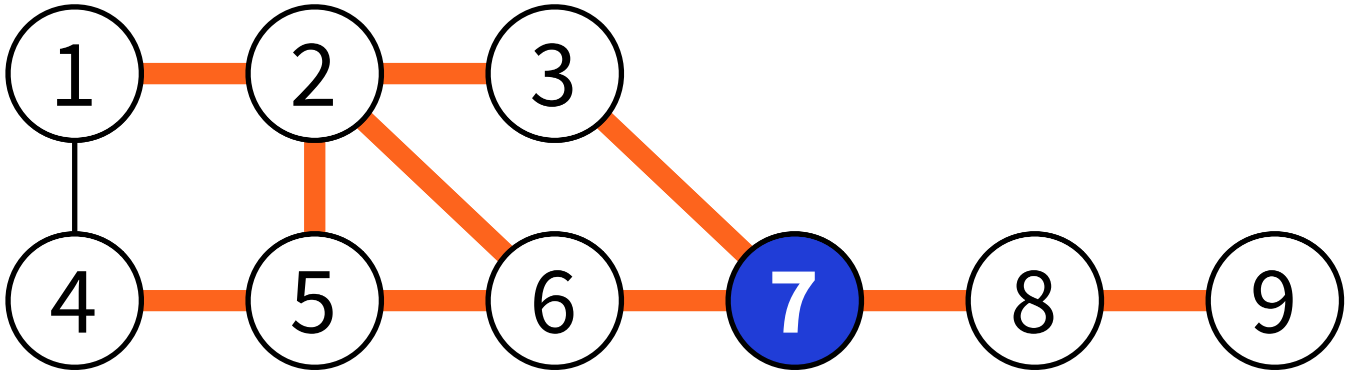

$E(7, 3)$

Gathering everything

- $E(v, r)$ = "edges within distance r from v"
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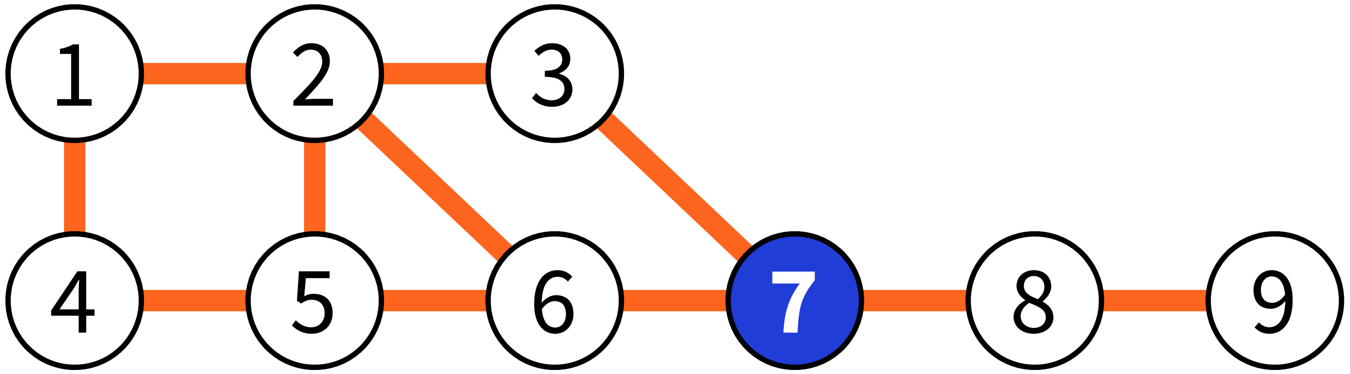
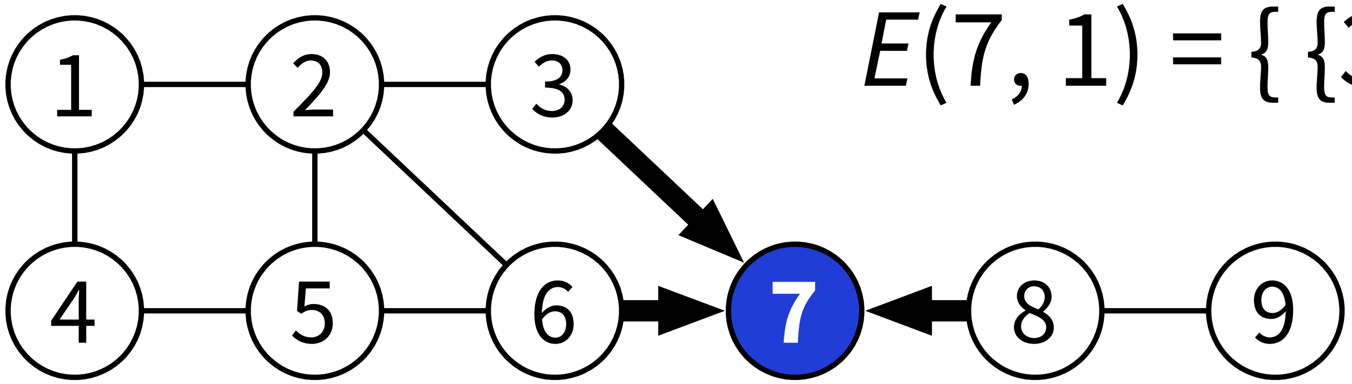


$E(7, 4)$

Gathering everything

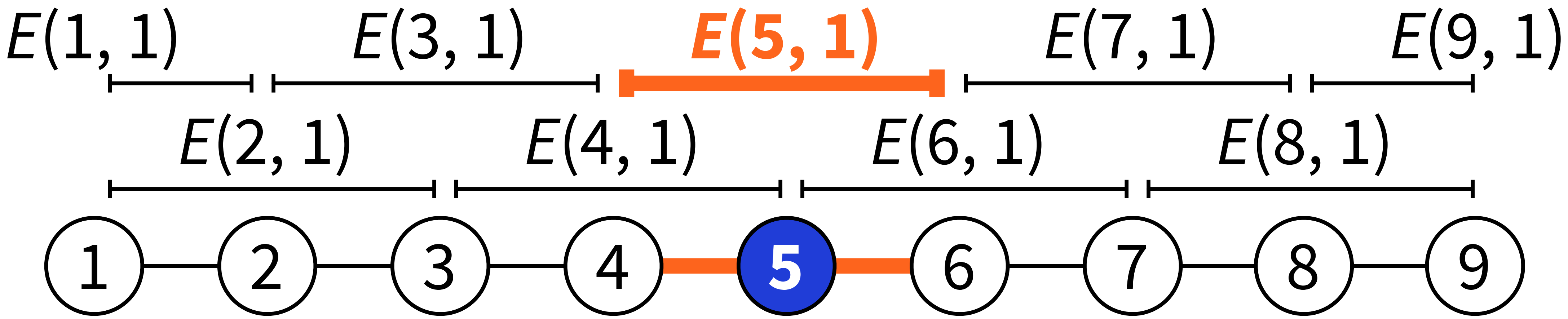
- **Each node *v* can learn *E*(*v*, 1) in 1 round**
  - send own ID to all neighbours



$E(7, 1) = \{ \{3, 7\}, \{6, 7\}, \{7, 8\} \}$

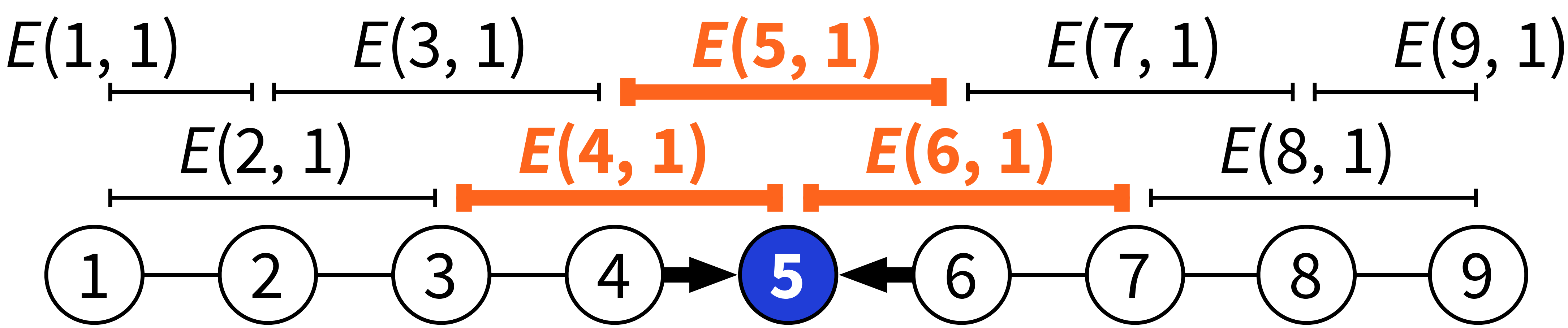
Gathering everything

- **Each node *v* can learn $E(v, 1)$ in 1 round**
  - send own ID to all neighbours



Gathering everything

- **Given $E(v, r)$, we can learn $E(v, r + 1)$ in 1 round**
  - send $E(v, r)$ to all neighbours, take union



Gathering everything

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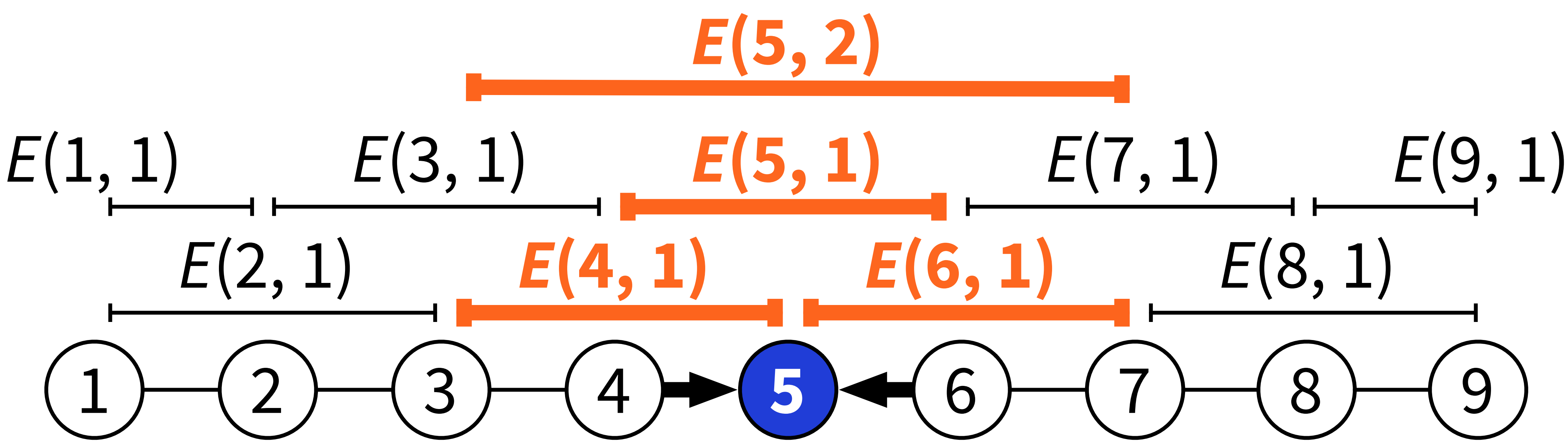


Gathering everything

- **One of the following holds:**
  - $E(v, r) \neq E(v, r + 1)$: learn something new
  - $E(v, r) = E(v, r + 1) = E$: we can stop
- **Proof idea:**
  - if $E(v, r) \neq E$, there are unseen edges adjacent to $E(v, r)$, and they will be in $E(v, r + 1)$

Example:
Graph colouring

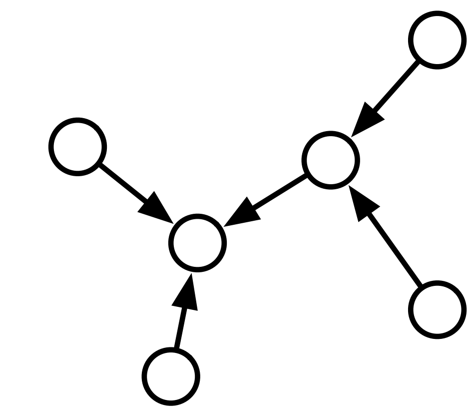
- **We can solve everything in $O(\mathrm{diam}(G))$ time**
- **What can be solved much faster?**
- **Example: graph colouring with $\Delta + 1$ colours**
  - can be solved in $O(\Delta + \log^* n)$ rounds
  - today: how to do it in $O(\Delta^2 + \log^* n)$ rounds?

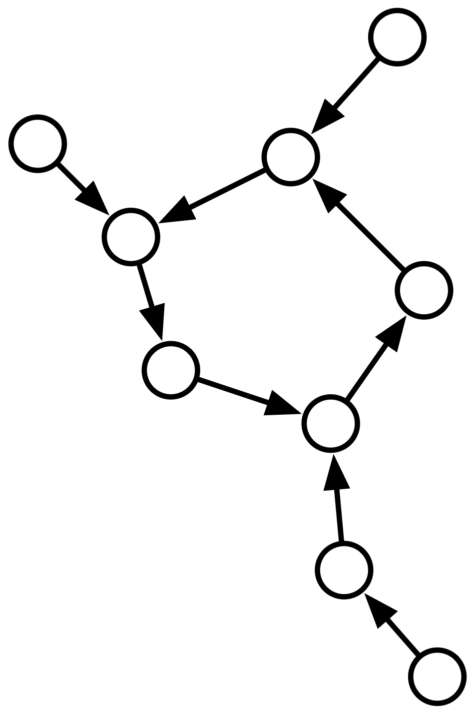
Example:
Graph colouring

- **Setting: LOCAL model, n nodes, any graph of maximum degree Δ**
- **We assume that n and Δ are known**
  - if not known: guess some n and Δ, colour what you can, increase n and Δ, …

Directed pseudoforest

- **Directed graph, outdegree ≤ 1**
- **Each node has at most one "successor"**
- **Easy to 3-colour in time $O(\log^* n)$, we will use this as subroutine**





Directed pseudoforest

- **Colouring directed pseudoforests almost as easy as colouring directed paths**
- **Recall path-colouring algorithm P3CBit…**

Algorithm P3CBit:
Fast colour reduction

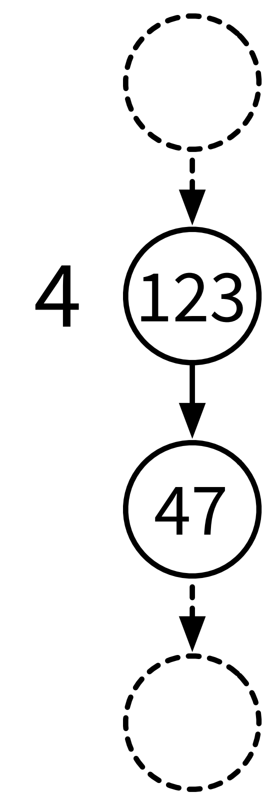
$\boldsymbol{c_0} = \mathbf{123} = \mathbf{01111\textcolor{orange}{0}11_2}$ (my colour)

$\boldsymbol{c_1} = \mathbf{47} = \mathbf{00101\textcolor{orange}{1}11_2}$ (successor's colour)

$\boldsymbol{i} = \mathbf{\textcolor{orange}{2}}$ (bits numbered 0, 1, 2, … from right)

$\boldsymbol{b} = \mathbf{\textcolor{orange}{0}}$ (in my colour bit number *i* was 0)

$\boldsymbol{c} = \mathbf{2 \cdot 2 + 0 = 4}$ (my new colour)



$k = 8$, reducing from $2^8 = 256$ to $2 \cdot 8 = 16$ colours

Directed pseudoforest

- **Colouring directed pseudoforests almost as easy as colouring directed paths**
- **Recall path-colouring algorithm P3CBit:**
  - nodes **only look at their successor**
  - my new colour ≠ successor's new colour
  - works equally well in directed pseudoforests!

Algorithm DPBit:
Fast colour reduction

$\boldsymbol{c_0} = \mathbf{123} = \mathbf{01111011_2}$ (my colour)

$\boldsymbol{c_1} = \mathbf{47} = \mathbf{00101111_2}$ (successor's colour)

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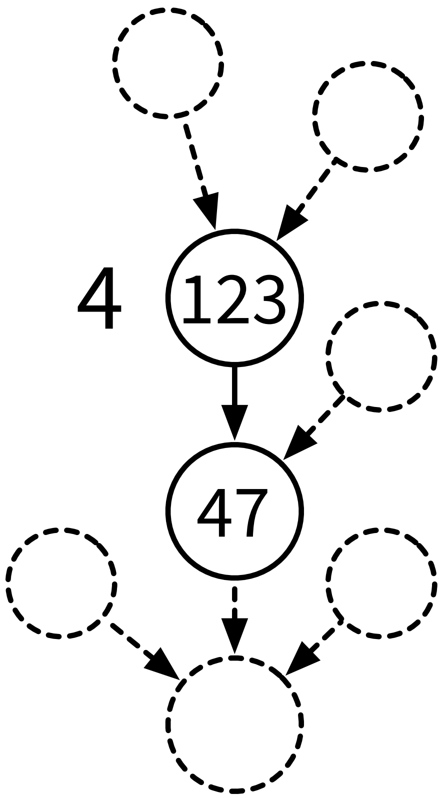

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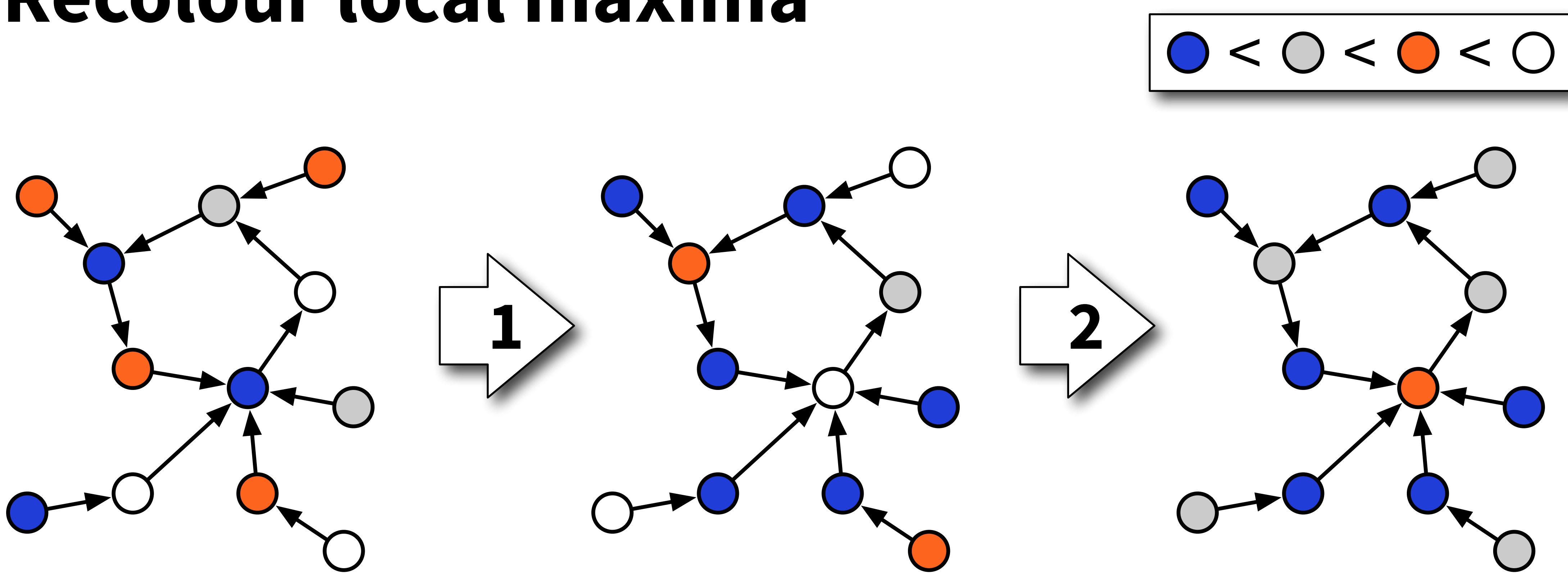
Directed pseudoforests

- **Unique identifiers = $n^{O(1)}$ colours**
- **Iterate DPBit for $O(\log^* n)$ steps to reduce the number of colours to 6**
- **Iterate DPGreedy for 3 steps to reduce the number of colours to 3**

Algorithm DPGreedy: **Slow colour reduction**

1. **Shift:** ***predecessors have the same colour***
2. **Recolour local maxima**



Directed pseudoforests

- Unique identifiers = $n^{O(1)}$ colours
- Iterate DPBit for $O(\log^* n)$ steps
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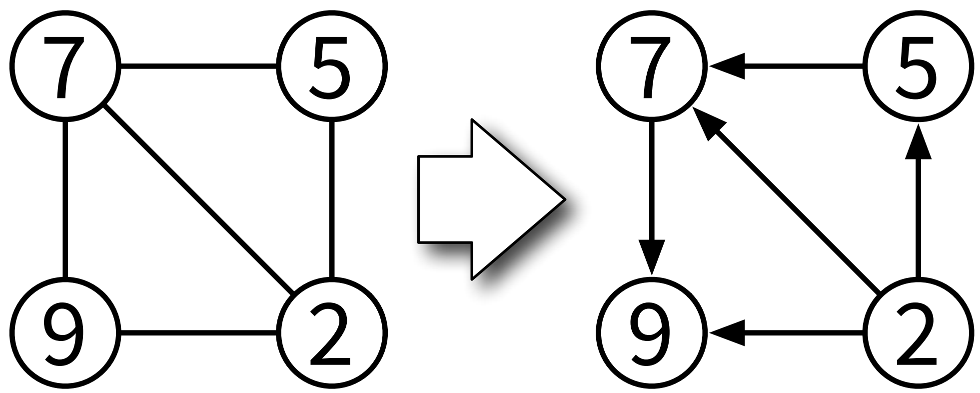
- Iterate DPGreedy for 3 steps
  to reduce the number of colours to 3

Algorithm BDColour: **Fast graph colouring**

- **Unique identifiers → orientation**
- **Port numbers → partition edges in Δ directed pseudoforests**
- **3-colour pseudoforests in time $O(\log^* n)$**
- **Merge pseudoforests in time $O(\Delta^2)$**

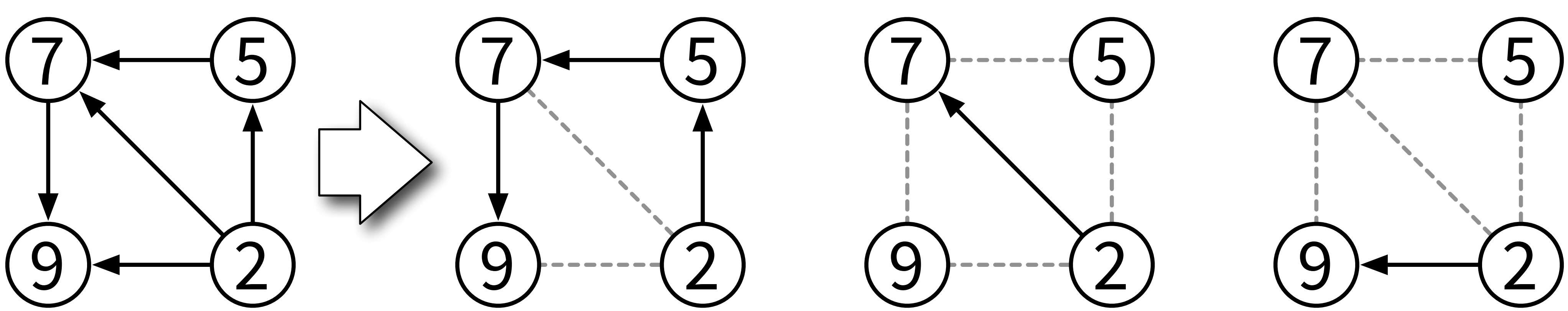
Algorithm BDColour:
Fast graph colouring

- **Unique identifiers → orientation**
  - edges directed from smaller to larger ID



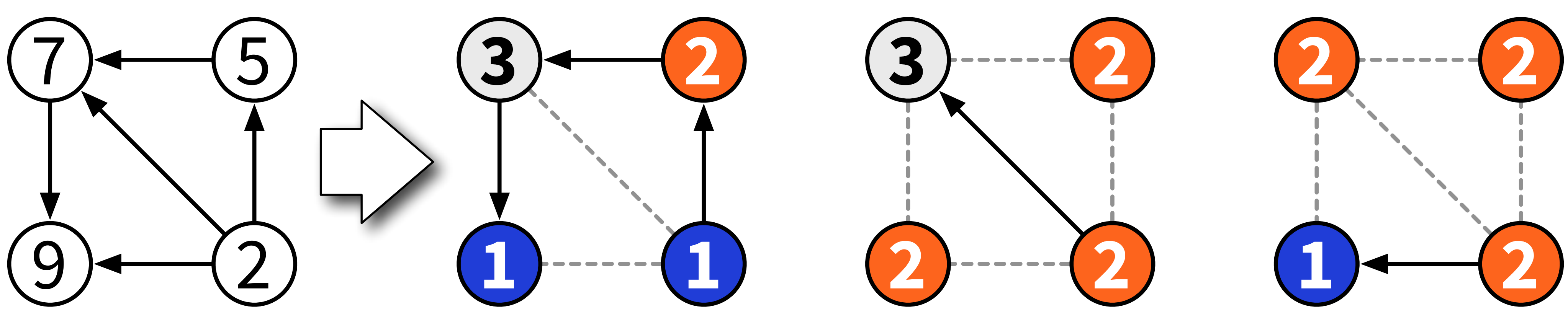
Algorithm BDColour:
Fast graph colouring

- **Port numbers → partition edges in Δ directed pseudoforests**
  - *k*th outgoing edge → *k*th pseudoforest



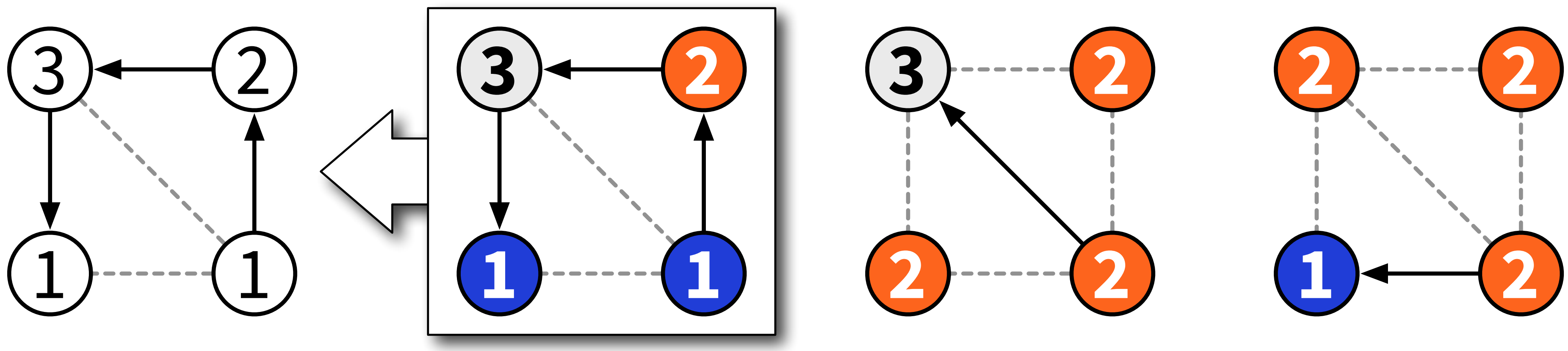
Algorithm BDColour: **Fast graph colouring**

- **3-colour pseudoforests in time $O(\log^* n)$**
  - all in parallel
  - each node has Δ roles



Algorithm BDColour: **Fast graph colouring**

- **Merge pseudoforests in time $O(\Delta^2)$**
  - maintain colouring with $\Delta + 1$ colours
  - add first forest: trivial



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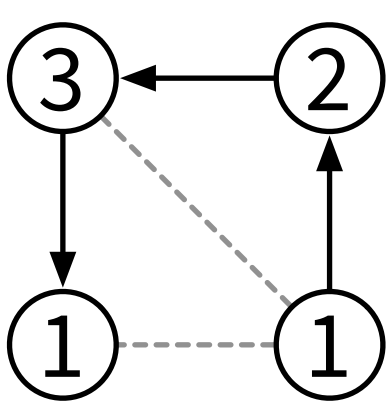

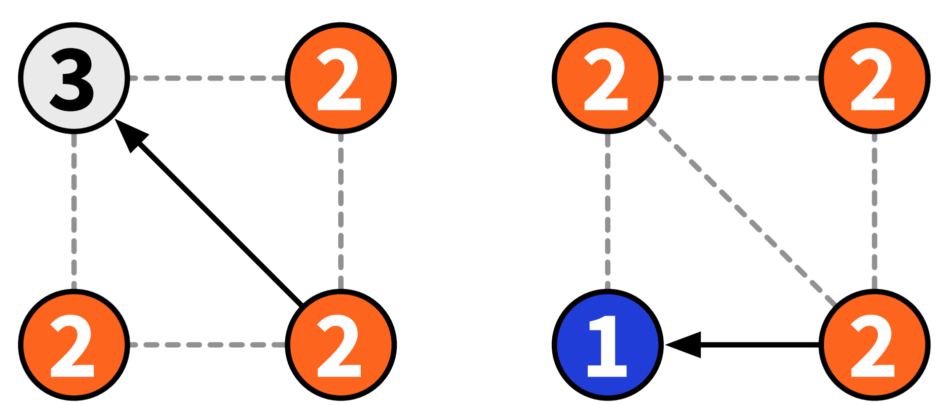

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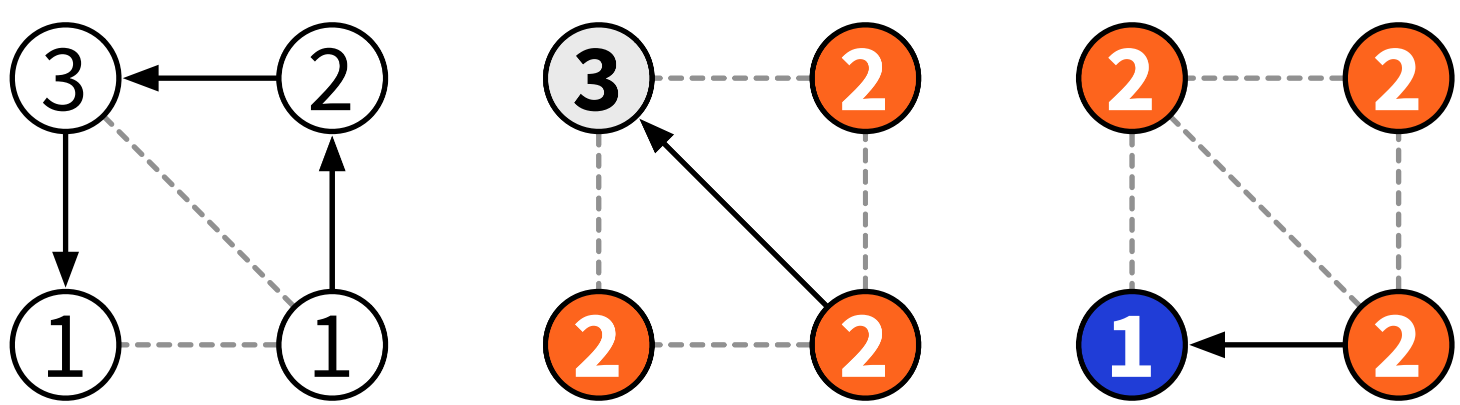

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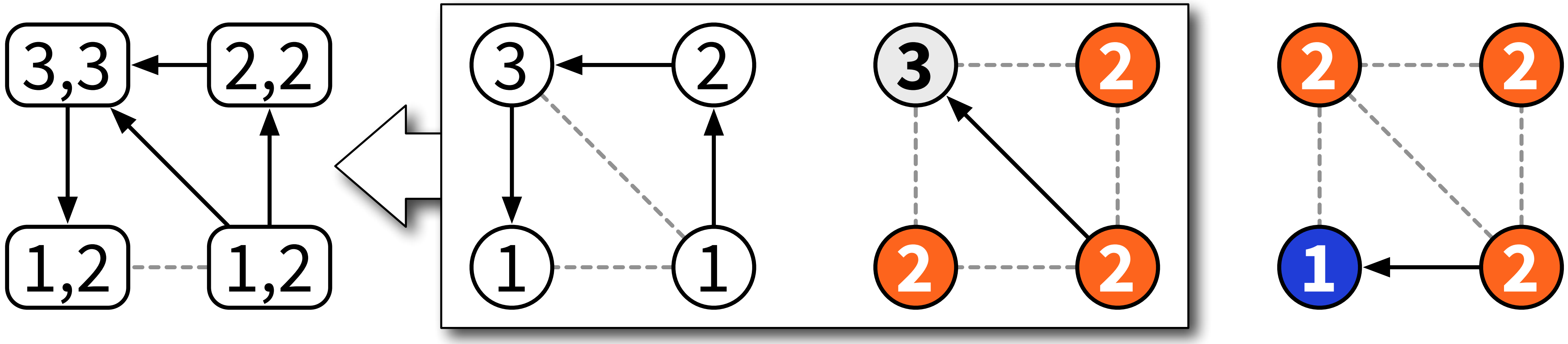

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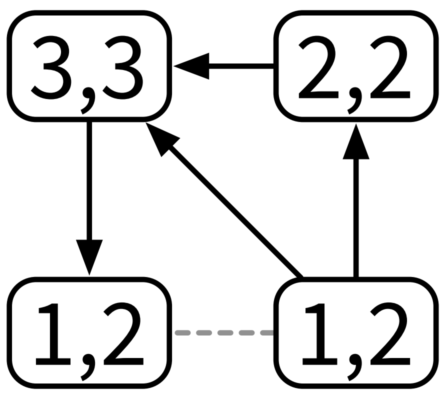
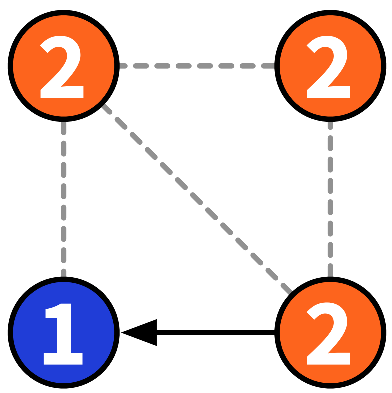
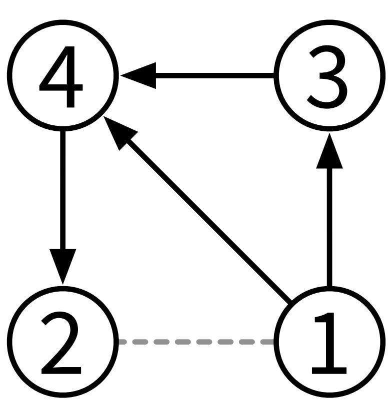
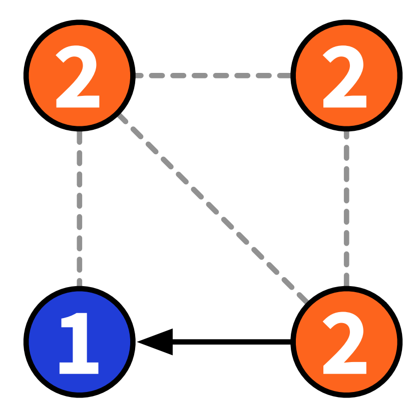




Algorithm BDColour: **Fast graph colouring**

- **Merge pseudoforests in time $O(\Delta^2)$**
  - maintain colouring with $\Delta + 1$ colours
  - add one forest → $3(\Delta + 1)$ colours → reduce





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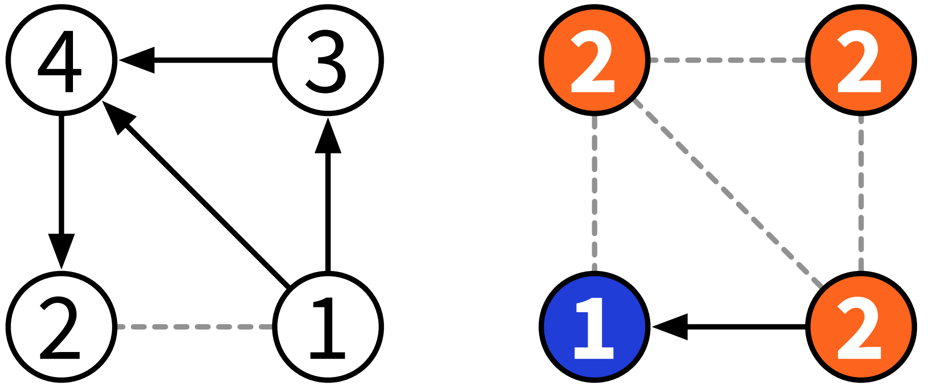

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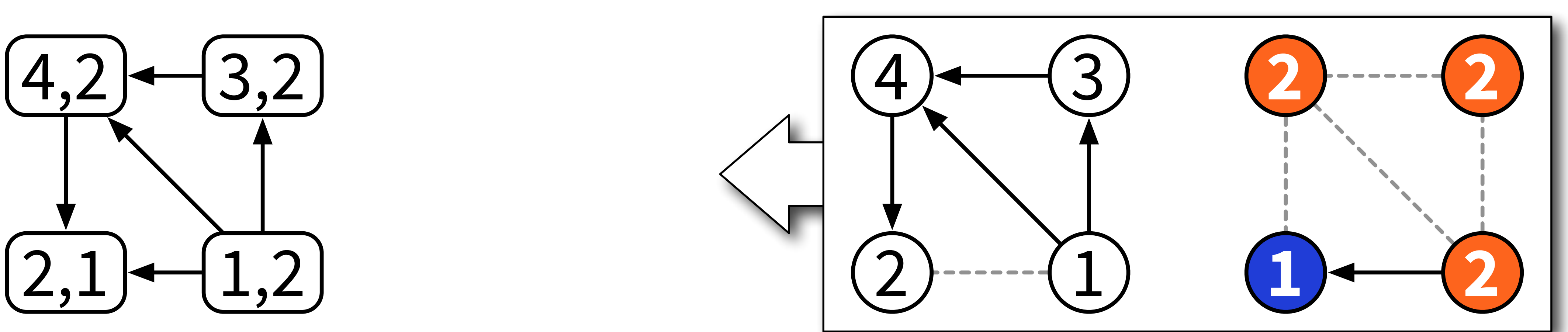

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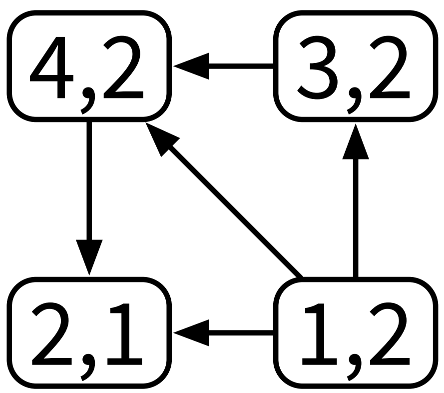

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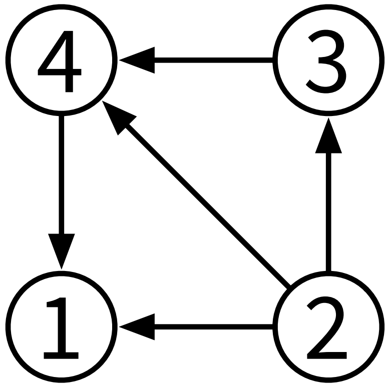

Algorithm BDColour:
Fast graph colouring

- **Merge pseudoforests in time $O(\Delta^2)$**
  - maintain colouring with $\Delta + 1$ colours
  - add one forest → $3(\Delta + 1)$ colours → reduce
- **Each merge + reduce takes $O(\Delta)$ rounds**
- **There are $O(\Delta)$ such steps**

Algorithm BDColour: **Fast graph colouring**

- **Unique identifiers → orientation**
- **Port numbers → partition edges in Δ directed pseudoforests**
- **3-colour pseudoforests in time $O(\log^* n)$**
- **Merge pseudoforests in time $O(\Delta^2)$**

Summary: **LOCAL model**

- **Unique identifiers**
- **Everything can be computed**
- **What can be computed fast?**
  - example: graph colouring

Summary: **LOCAL model**

- **Unique identifiers**
- **Everything can be computed**
  - cheating with large messages
  - what if we can only use small messages?
  - this is covered next week…

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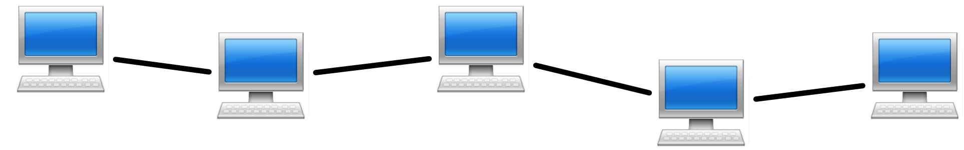
  - network = path 
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